UNIT P6
Problem Solving
Upper Primary

Blake's Topic Bank

Problem Solving
Using Simpler Numbers

by Sharon Shapiro

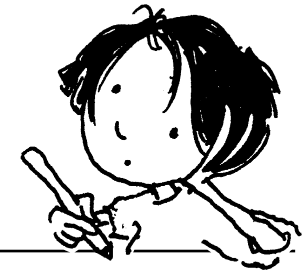
This unit contains:

- Teaching notes
- 3 teaching examples
- 1 BLM
- 18 task cards
- Answers

Problem Solving
Using Simpler Numbers

Sharon Shapiro

Upper Primary



The Problem Solving Process

It is important that students follow a logical and systematic approach to their problem solving. Following these four steps will enable students to tackle problems in a structured and meaningful way.

Step 1: Understanding the problem

- ❖ Encourage students to read the problem carefully a number of times until they fully understand what is wanted. They may need to discuss the problem with someone else or rewrite it in their own words.
- ❖ Students should ask internal questions such as, what is the problem asking me to do, what information is relevant and necessary for solving the problem.
- ❖ They should underline any unfamiliar words and find out their meanings.
- ❖ They should select the information they know and decide what is unknown or needs to be discovered. They should see if there is any unnecessary information.
- ❖ A sketch of the problem often helps their understanding.

Step 2: Students should decide on a strategy or plan

Students should decide how they will solve the problem by thinking about the different strategies that could be used. They could try to make predictions, or guesses, about the problem. Often these guesses result in generalisations which help to solve problems. Students should be discouraged from making wild guesses but they should be encouraged to take risks. They should always think in terms of how this problem relates to other problems that they have solved. They should keep a record of the strategies they have tried so that they don't repeat them.

Some possible strategies include:

- ❖ Drawing a sketch, graph or table.
- ❖ Acting out situations, or using concrete materials.
- ❖ Organising a list.
- ❖ Identifying a pattern and extending it.
- ❖ Guessing and checking.
- ❖ Working backwards.
- ❖ Using simpler numbers to solve the problem, then applying the same methodology to the real problem.
- ❖ Writing a number sentence.
- ❖ Using logic and clues.
- ❖ Breaking the problem into smaller parts.

Step 3: Solving the problem

- ❖ Students should write down their ideas as they work so they don't forget how they approached the problem.
- ❖ Their approach should be systematic.
- ❖ If stuck, students should reread the problem and rethink their strategies.
- ❖ Students should be given the opportunity to orally demonstrate or explain how they reached an answer.

Step 4: Reflect

- ❖ Students should consider if their answer makes sense and if it has answered what was asked.
- ❖ Students should draw and write down their thinking processes, estimations and approach, as this gives them time to reflect on their practices. When they have an answer they should explain the process to someone else.
- ❖ Students should ask themselves 'what if' to link this problem to another. This will take their exploration to a deeper level and encourage their use of logical thought processes.
- ❖ Students should consider if it is possible to do the problem in a simpler way.

Teaching Notes Using Simpler Numbers

When attempting to solve a difficult or complicated problem it can be a useful strategy to begin by solving a simpler problem of the same type. Once a method for solving this type of problem has been established, a solution for the more difficult example can be worked out.

One way of applying this strategy is to replace larger numbers in the problem with smaller numbers, so that the calculations are easier. It may be a good idea to attempt several simpler problems before moving back to the original difficult problem and applying the same method for solving it.

If a series of simpler problems is solved, often a pattern will emerge which can then be applied to more complicated problems. This pattern can often be plotted in a table or expressed as a diagram. Having a visual representation makes it easier to follow the pattern through to reach the solution for the more difficult problem.

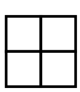
The strategy of using simpler numbers can also be used in conjunction with other problem solving strategies, as it is helpful in solving difficult problems of almost any type.

In order to use this strategy effectively, students will need to develop the following skills and understandings.

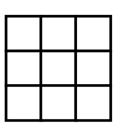
BREAKING DOWN SPATIAL PROBLEMS

Often students will be asked to work out how many squares appear in a large grid, for example a chessboard. The most effective way to solve these problems is to calculate how many squares appear in a number of smaller grids and to establish the pattern. A drawing of the grid will be very helpful. The pattern can then be recorded in a table and used to work out how many squares appear in a larger grid. (For more details see Teaching Example 3.)

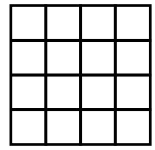
A 2 x 2 board has:
four 1 x 1 squares
and one 2 x 2 square.



A 3 x 3 board has:
nine 1 x 1 squares,
four 2 x 2 squares
and one 3 x 3 square.



A 4 x 4 board has:
sixteen 1 x 1 squares,
nine 2 x 2 squares,
four 3 x 3 squares and
one 4 x 4 square.



A pattern begins to emerge.

SIMPLIFYING NUMERICAL PROBLEMS

Complicated numerical problems may be very difficult for students to solve. To make these problems easier, students should be encouraged to begin by using simpler numbers to work out a solution. By doing this they will gain confidence and feel more comfortable about tackling problems of this kind. Also, in many cases, by working through a series of simpler problems a pattern will emerge that can be used to solve problems with harder numbers.

Example 1: How many times will the digit seven appear as part of a number, when counting all the numbers from 1 to 374?

Work out how many times seven appears from 0 to 10, and then how many times from 10 to 20. Once you have worked out the pattern for the lower numbers, you will be able to calculate how many sevens appear in the higher numbers and so reach a solution. (For more details see Teaching Example 1.)

Example 2: If nine people complete some work in six hours how long will it take five people to do the same amount of work?

Begin by rewording the problem in a simpler way, for example, if it took three people five hours how long would it take two people.

Work out how long it would take one person to complete the work by multiplying three by five. One person would take 15 hours, so two people would take seven and a half hours. Now that you have established a method for solving this type of problem you can approach the more difficult one.

Again, work out how long it would take one person to do the task and then divide the total number of hours by five people. Therefore, if it takes nine people six hours, you need to multiply six hours by nine to find out how long it would take one person. Then divide this total figure by five to find out how long it would take five people. (For more details see Teaching Example 2.)

EXAMPLE 1

If the houses in your street are numbered from 1 to 150, how many houses will have the digit eight as part of the number?

Understanding the problem

WHAT DO WE KNOW?

The houses are numbered from 1 to 150.
Some of the houses have an eight in their number.

WHAT DO WE NEED TO FIND OUT?

Questioning:
How many houses have an eight as part of their number?

Planning and communicating a solution

Students should begin by working out how many of the numbers from 1 to 10 have an eight in them. Then they should calculate how many of the numbers from 10 to 20 have an eight in them. It might be helpful to write the results as part of a table.

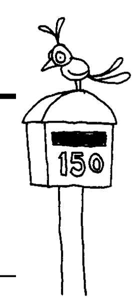
Students will find that the eight appears once in every group of ten consecutive numbers. From 1 to 150 there are fifteen groups of ten. These eights all appear as the first digit of the number, or in the ones place.

Some numbers from 1 to 150 also have an eight as the second digit, in the tens place. How many eights are in the tens place?

All the numbers from 80 to 89 have an eight in the tens place. There are ten numbers from 80 to 89, but 88 is not counted as it was already included in the total because it has an eight in the ones place.

House numbers	Eights in ones place	Eights in tens place	Eights in 100s place
1 – 9	1 (8)	0	0
10 – 19	1 (18)	0	0
20 – 29	1 (28)	0	0
30 – 39	1 (38)	0	0
and so on	48, 58, 68, 78)		
80 – 89	1 (88)	9 extra numbers (80, 81, 82, 83, 84, 85, 86, 87, 89)	0
90 – 99	1 (98)	0	0
100 – 109	1 (108)	0	0
110 – 119	1 (118)	0	0
and so on	128, 138		
140 – 149	1 (148)	0	0

Therefore, the numbers 1 to 150 have 15 + 9 = 24 eights.



Reflecting and generalising

By breaking the problem down into segments, it becomes manageable and easy to calculate. Also, by recording the results in a table it was possible to keep track of answers and identify a pattern which could be used to solve the whole problem.

Extension

Ask students to solve problems involving a larger spread of numbers.
Challenge them to find out how often two digits will appear in a sequence of numbers, for example, three and one.

EXAMPLE 2

If it takes 15 children ten hours to complete half of a group project, how long will it take four children to complete the other half?

Understanding the problem

WHAT DO WE KNOW?

Fifteen children complete half the project in ten hours.

WHAT DO WE NEED TO FIND OUT?

Questioning:
How long will it take four children to complete the other half?

Planning and communicating a solution

We start by taking a simpler example: If it takes two children ten hours to complete a task, how long will it take four children?

First we have to find out how long it will take one child to complete the task.

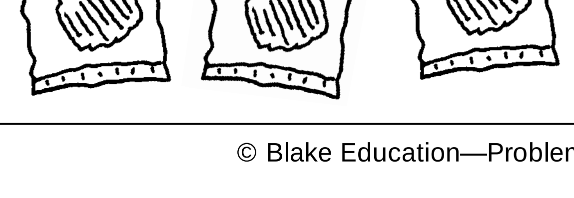
If two children take ten hours, then one child would take 20 hours.
2 x 10 = 20 hours

How long would it take four children?
20 ÷ 4 = 5
Four children would take five hours.

Now that you have worked out the solution for a simpler problem you can attempt the real problem. Again start by working out how long it would take one child to complete the work.

15 children complete their half of the task in ten hours.
One child would complete half the task in 150 hours.
10 x 15 = 150 hours

Therefore, four children would need 37.5 hours to complete the other half of the task.
150 ÷ 4 = 37.5 hours

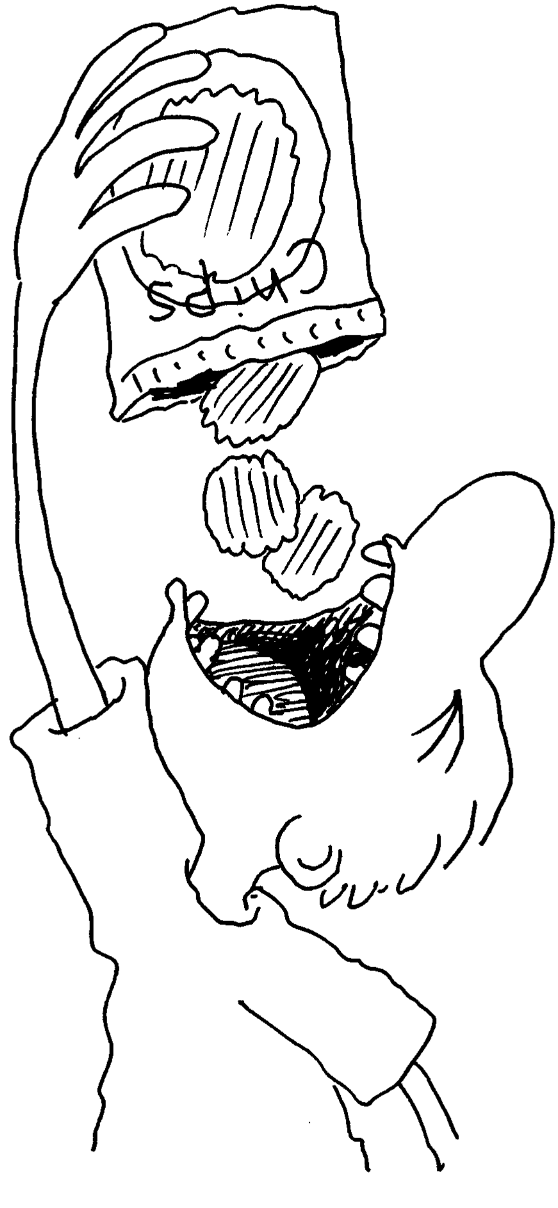


Reflecting and generalising

By working through the task using simpler numbers, the calculations were less difficult and a solution easier to find. A similar pattern was followed when working with the larger numbers.

Extension

If 25 children take two hours to eat 250 packets of chips, and 20 children take three hours, who eats more quickly – the children in the first group or the second group? How many packets does each child in each group eat? Ask students to write their own problems and then to compare their results.



Example 3

A game uses a board with a 6 x 6 square grid. How many squares of all sizes are on the board?

Understanding the problem

What do we know?

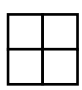
The board is a 6 x 6 grid.
There are squares of varying sizes, eg 1 x 1, 2 x 2.

What do we need to find out?

Questioning:
How many squares of all sizes can be seen in the board?

Planning and communicating a solution

Begin by counting out how many squares of different sizes can be found on smaller sized grids. Start with a 2 x 2 grid, then a 3 x 3 grid and so on. It is helpful to draw the grids so you can easily see and count how many squares there are.



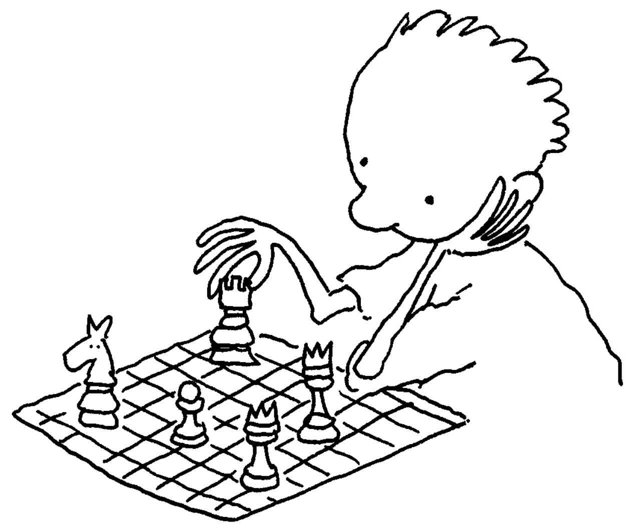
Once you have worked out how many squares are in several smaller grids you should write your results in a table.

Reflecting and generalising

By starting with a small area it was easy to count the number of squares in different sizes. By recording these results in a table it was possible find the pattern and to make sure no squares were missed out. A logical and methodical approach was used.

Extension

How many squares of all sizes would be in a 10 x 10 grid? How many squares of all sizes are there in a variety of rectangular grids? Do they follow the same pattern as square grids?

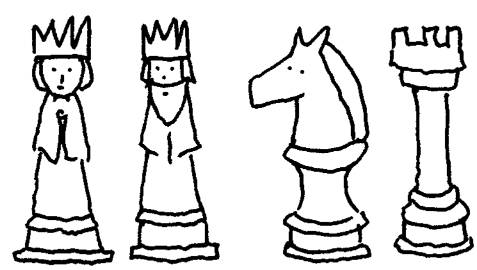


Size of board	Number of squares						
	1 x 1	2 x 2	3 x 3	4 x 4	5 x 5	6 x 6	Total
2 x 2	4	1					5
3 x 3	9	4	1				14
4 x 4	16	9	4	1			30
5 x 5	25	16	9	4	1		55
6 x 6	36	25	16	9	4	1	91

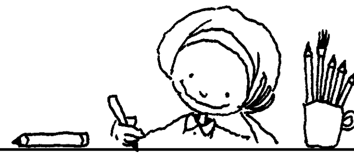
A pattern begins to emerge, 1 + 4 + 9 + 16 + 25...

The pattern can be carried on in the same way to find the number of squares in any size grid.

There are 91 squares of all sizes in a 6 x 6 grid.
1 + 4 + 9 + 16 + 25 + 36 = 91



BLM Using Simpler Numbers



★ Understanding the problem

What do you know? List the important facts from the problem.

What do you need to find out?
What is the problem asking you to do? What are you uncertain about? Do you understand all aspects of the problem? Is there any unfamiliar or unclear language?

★ Planning and communicating a solution

Start by reading the problem and deciding what can be simplified. Decide where to start? Is a table necessary? Do you need to draw a grid? Look at a small section first and extend gradually. Can you see a pattern?

★ Reflecting and generalising

Did the strategy work as planned? Will you be able to apply this method of problem solving to other similar problems? Could you have used a different method to solve the problem?

★ Extension

How can this strategy be applied to more complicated figures involving additional factors?

Problem 1

Number 123

Level 1

At Christmas, six friends decide to give each other presents? How many presents are given altogether?

Problem 2

Number 123

Level 1

A group of children are playing on the beach. They are hopping and jumping along the beach. Each child does three hops followed by one jump. From a standing start, how many footprints in the sand would there be for each child after –

3 jumps?
10 jumps?
20 jumps?

Problem 3

Number 123

Level 1

At a school reunion, all of the 12 people present give their address to every other person there. How many addresses will be given out?

Problem 4

Number 1 2 3

Level 1

Marco is making a fence out of match sticks for his toy farm. The fence is made up of a series of square sections. How many matches will Marco need to make a fence five sections long?

How many matches will he need for –
8 sections?
20 sections?
30 sections?

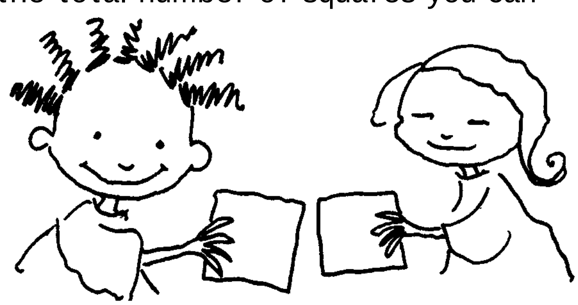
Problem 5

Number 1 2 3

Level 1

James and Ling are making up a board game. The board for their game is three squares by three squares in size. What is the total number of squares you can find on their game board?

If they increased the size of the board to four squares by four squares, what would be the total number of squares on the board? What if they increased the board to five by five squares?

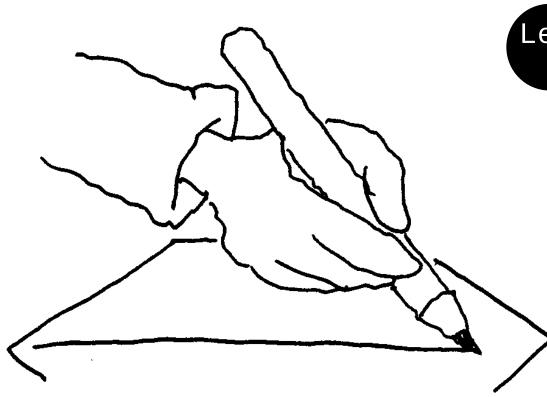


Problem 6

Space

Level 2

How many diagonals can be drawn from one corner of the following regular two dimensional shapes – a square; a pentagon; hexagon; heptagon; octagon; a regular flat shape with 50 sides; 80 sides; or 100 sides?

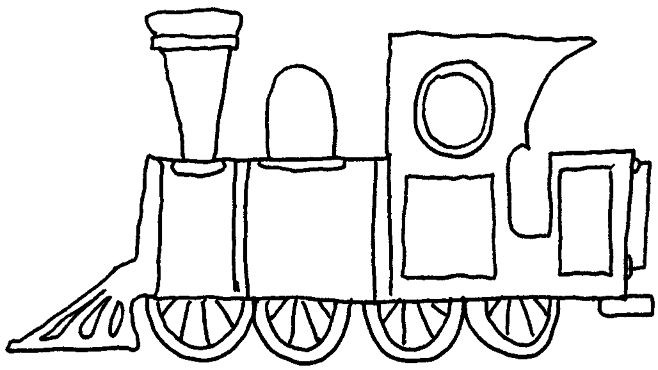


Problem 7

Measurement

Level 2

At exactly the same time, two trains depart in opposite directions from the station. One train is moving at a speed of 56 kilometres per hour, while the other is moving at a speed of 64 kilometres per hour. After three hours how far apart will they be?



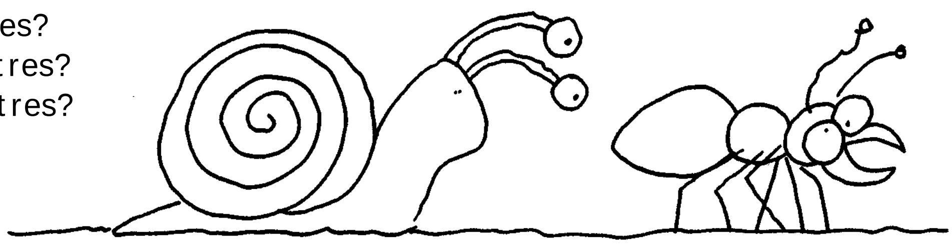
Problem 8

Measurement

Level 2

A snail and an ant are moving slowly crossing the playground. For every ten metres the ant travels, the snail travels one metre. By how far will the ant be ahead of the snail over a distance of –

a. 10 metres?
b. 100 metres?
c. 200 metres?

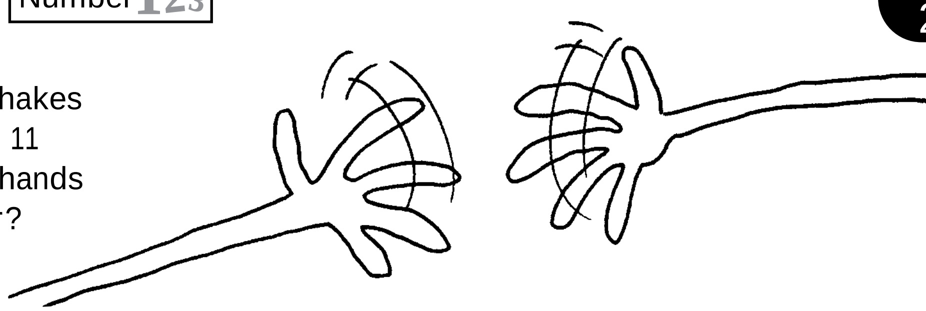


Problem 9

Number 123

Level 2

How many handshakes will take place if 11 people all shake hands with one another?



PROBLEM SOLVING TASK CARDS - Using Simpler Numbers

Problem 10

Number 123

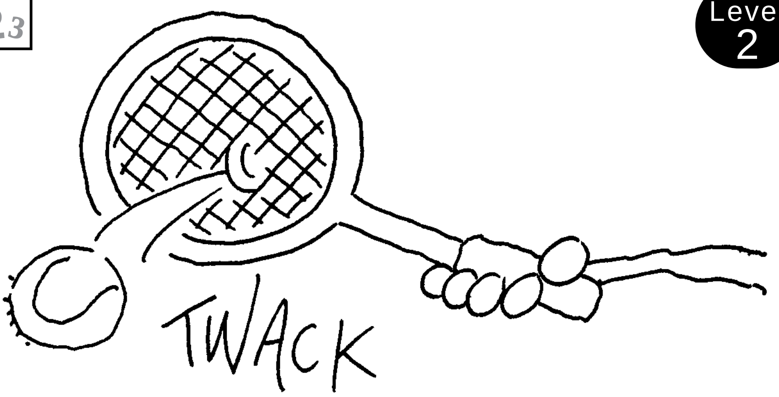
Level 2

If nine fours are all multiplied together (for example, 4 x 4 x 4 x 4...) what is the remainder when the answer is divided by six.

Problem 11

Number 123

Level 2



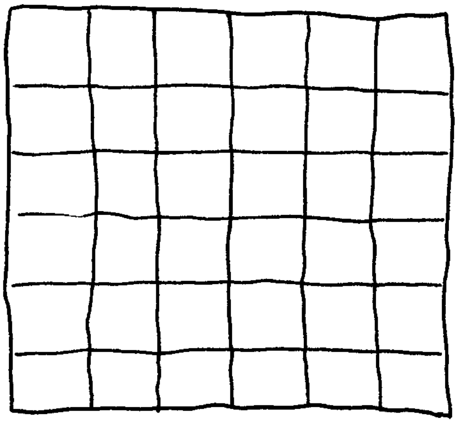
Twelve people are playing in a tennis competition. Each person must play one game against each other person. How many games are played altogether?

Problem 12

Number 123

Level 2

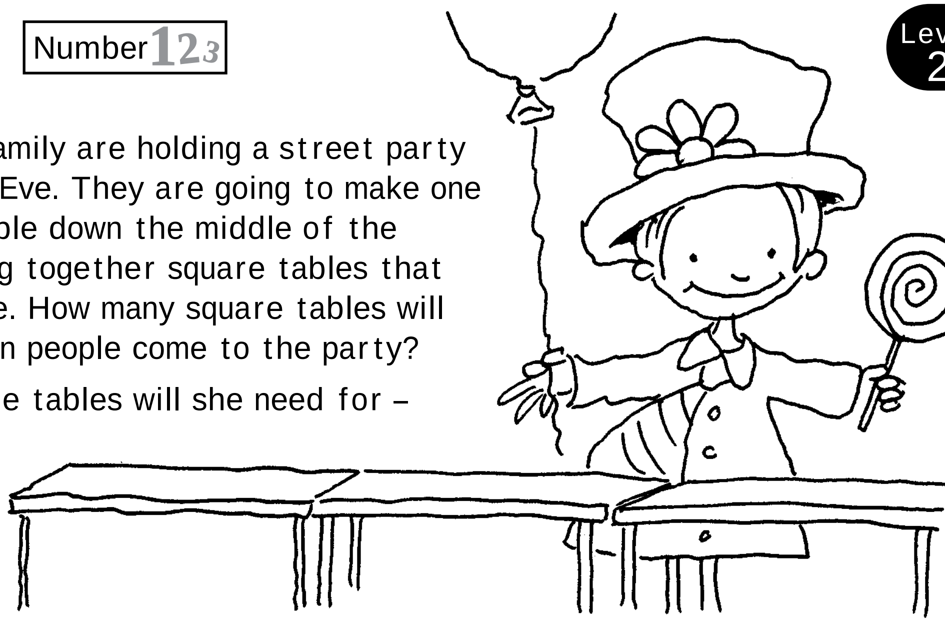
On the playground is painted a grid of six squares by six squares. The teacher has asked her class to place blue markers in all of the 1 x 1 squares, red markers in the 2 x 2 squares, green markers in the 3 x 3 squares, purple in the 4 x 4 squares, orange in the 5 x 5 squares and yellow markers in the 6 x 6 squares. How many markers of each colour will the class need? How many markers will they use altogether?



Problem 13

Number 123

Level 2



Mary and her family are holding a street party for New Year's Eve. They are going to make one long banquet table down the middle of the street by joining together square tables that seat four people. How many square tables will Mary need if ten people come to the party?

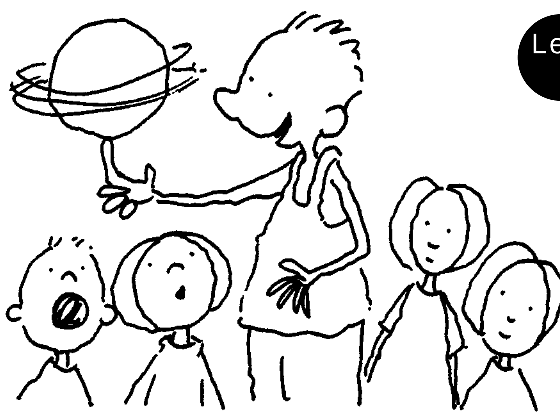
How many square tables will she need for –

30 people?
50 people?
100 people?

Problem 14

Number 123

Level 2



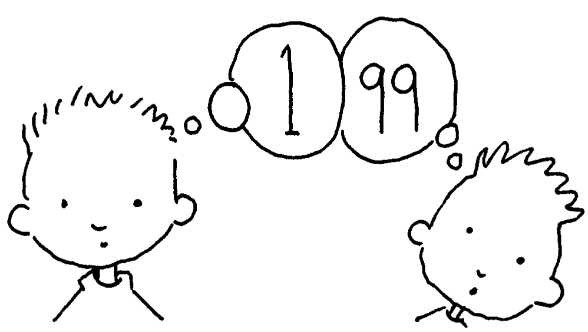
A basketball team of five players must choose a captain and a vice captain. How many different combinations are possible?

How many combinations are possible for a football team of 12 players?

Problem 15

Number 123

Level 3



Calculate the sum of every odd number between one and 99,
1 + 3 + 5 + ... + 97 + 99.

Problem 16

Number 123

Level 3

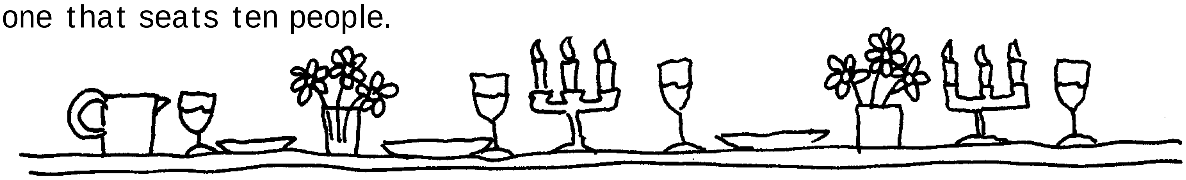
At a restaurant there are 40 square tables and each can seat four people. When two tables are joined together they can seat six. When three tables are joined together they can seat eight.

The following parties have made bookings at the restaurant:

Thursday a party of 64; Friday a group of 98 and Saturday a group of 120.

How can the restaurant organise the tables to fit each of the parties in easily?

Note: the restaurant is quite small and the largest group of tables it can fit is one that seats ten people.



Problem 17

Number 123

Level 3

A group of people has been stranded on a farm by floods. There are 40 people on the farm and they have enough food to last for 12 days. Three days later eight more people struggle to reach them from a neighbouring farm. How long will the rest of the food last now that there are eight extra people to feed?

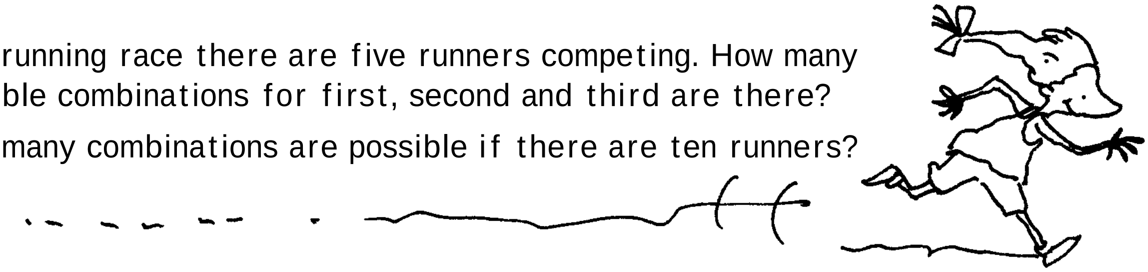
Problem 18

Number 123

Level 3

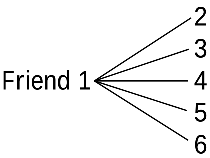
In a running race there are five runners competing. How many possible combinations for first, second and third are there?

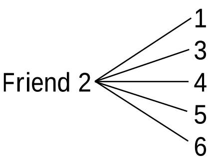
How many combinations are possible if there are ten runners?



Answers to Task Cards

Problem 1





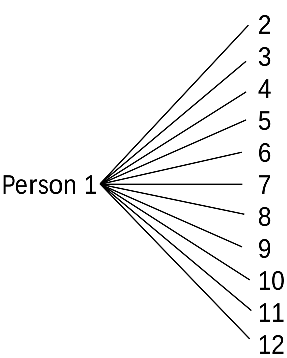
A pattern becomes clear, showing that each of the six friends has to give five presents, so 6 x 5 = 30.

Problem 2

Each child starts with two footprints where they are standing when they begin. Every set of three hops and one jump creates five footprints. So for three jumps, 2 + 3 x 5 = 17 footprints.

10 jumps, 2 + 10 x 5 = 52 footprints
20 jumps, 2 + 20 x 5 = 102 footprints

Problem 3



Start by working out how many addresses one person gives out. A pattern emerges showing that each of the people at the school reunion gives out 11 addresses so, 12 x 11 = 132 addresses are given out.

Problem 4

For the first section Marco uses four matches, but for each subsequent section he only uses three matches. A pattern emerges –
1 section, 1 x 3 + 1 = 4
2 sections, 2 x 3 + 1 = 7
3 sections, 3 x 3 +1 = 10
So to work out the number of matches used in any number of sections, count the number of sections then multiply that number by three and add one.
5 sections, 5 x 3 + 1 = 16
8 sections, 8 x 3 + 1 = 25
20 sections, 20 x 3 +1 = 61
30 sections, 30 x 3 = 91

Problem 5

	Number of squares					
Size of board	1 x 1	2 x 2	3 x 3	4 x 4	5 x 5	Total
2 x 2	4	1				5
3 x 3	9	4	1			14
4 x 4	16	9	4	1		30
5 x 5	25	16	9	4	1	55

Problem 6

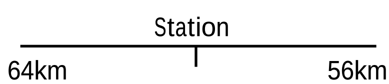
A square: you can draw one diagonal from one corner of a square.
A pentagon: two diagonals.
A hexagon: three diagonals.
A heptagon: four diagonals.
An octagon: five diagonals.

A pattern emerges showing that the number of diagonals you can draw from one corner equals the number of sides minus three.

Therefore:
A flat figure with 50 sides: 47 diagonals.
A flat figure with 80 sides: 77 diagonals.
A flat figure with 100 sides: 97 diagonals.

Problem 7

After one hour:



64 + 56 = 120 km

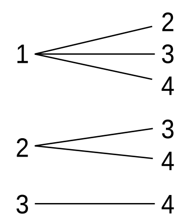
After 3 hours:
120 km x 3 = 360 km apart.

Problem 8

After 10 metres the ant is nine metres ahead.
After 100 metres the ant is 90 metres ahead.
After 200 metres the ant is 180 metres ahead.

Problem 9

Begin by working out how many handshakes take place between four people:



Six handshakes in all for four people.

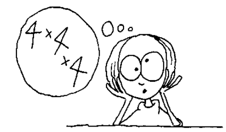
If there are four people, the first person has to shake hands with three other people, but the second person only has to shake hands with two other people. This pattern can be extended to work out how many handshakes take place for 11 people.

The first person shakes ten hands, the second person shakes nine hands and so on. 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 55. There are 55 handshakes if 11 people each shake every other person's hand.

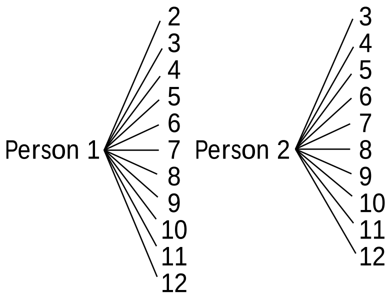
Problem 10

2 fours, 4 x 4 ÷ 6 = 2 r 4
3 fours, 4 x 4 x 4 ÷ 6 = 10 r 4
4 fours, 4 x 4 x 4 x 4 ÷ 6 = 42 r 4
5 fours, 4 x 4 x 4 x 4 x 4 ÷ 6
= 170 r 4

A pattern emerges showing that the remainder in each case is four so the remainder for nine fours divided by six will also be four.



Problem 11



A pattern emerges showing that Person 1 plays 11 games, and then each person after that plays one less game.

11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 66

After 66 games each person will have played one game against each other person.

Problem 12

Blue markers = 36
Red markers = 25
Green markers = 16
Purple markers = 9
Orange markers = 4
Yellow markers = 1

Total number of markers = 91

(See answer to problem 5 for more details.)

Problem 13

Begin by using a smaller number of guests and tables to work out a pattern.

1 table fits 4 people
2 tables joined fit 6 people
3 tables fit 8 people
4 tables fit 10 people.

A pattern emerges showing that if you multiply the number of tables by two and add two more you find how many people can fit at the tables.

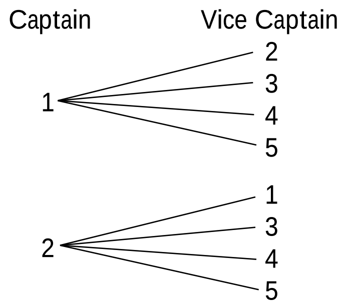
So, for 30 people you will need 14 tables.
14 x 2 +2 = 30

To work backwards you can minus two from the number of people you have and divide this number by two.

50 – 2 = 48
48 ÷ 2 = 24
So for 50 people you need 24 tables.

100 – 2 = 98
96 ÷ 2 = 49
So for 100 people you need 49 tables.

Problem 14



For each of the five players there are four different combinations, so there are 20 combinations in all.

For a team of 12 there are 12 x 11 = 132 combinations.

Problem 15

Being by adding less numbers together.

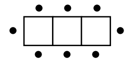
Numbers	Total
1	1
1 + 3	4
1 + 3 + 5	9
1 + 3 + 5 + 7	16
1 + 3 + 5 + 7 + 9	25

A pattern emerges showing that the answer is always the square of how many numbers are being added together. So if four numbers are being added, the answer is the square of four, 4 x 4 = 16.

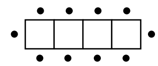
There are 50 odd numbers from one to 99, so the answer will be the square of 50.
50 x 50 = 2500.

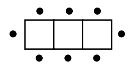
Problem 16

There are a number of correct answers for this problem.
On Thursday the tables could be arranged to have eight lots of three tables joined together, 8 x 8 = 64

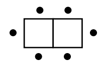
 x 8

Friday, nine lots of four tables joined together, plus one lot of three tables joined together, 9 x 10 + 8 = 98.

 x 9

 x 1

Saturday, 20 lots of four tables joined together, 20 x 6 = 120.

 x 20

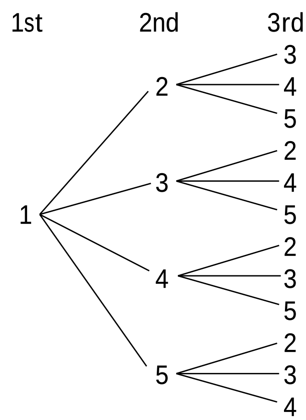
Problem 17

If the food will last 40 people for 12 days, after three days have passed the food will last the 40 people nine days.

So that food would last one person, 40 x 9 = 360 days.

When the eight extra people arrive there are 48 people to feed so 360 ÷ 48 = $7\frac{1}{2}$ days.

Problem 18



Starting by assuming that runner one was the winner and work through all the options for first, second and third place. For a race with five runners there are 12 different combinations for each runner, so 12 x 5 = 60. For a race with ten runners there are 72 different combinations for each runner so 10 x 72 = 720.