



Problem Solving

Creating a Tree Diagram

by Sharon Shapiro



This unit contains:

- Teaching notes
- 3 teaching examples
- 1 BLM
- 18 task cards
- Answers

Problem Solving

Creating a Tree Diagram

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Upper Primary



THE PROBLEM SOLVING PROCESS

It is important that students follow a logical and systematic approach to their problem solving. Following these four steps will enable students to tackle problems in a structured and meaningful way.

STEP 1: UNDERSTANDING THE PROBLEM

- ❖ Encourage students to read the problem carefully a number of times until they fully understand what is wanted. They may need to discuss the problem with someone else or rewrite it in their own words.
- ❖ Students should ask internal questions such as, what is the problem asking me to do, what information is relevant and necessary for solving the problem.
- ❖ They should underline any unfamiliar words and find out their meanings.
- ❖ They should select the information they know and decide what is unknown or needs to be discovered. They should see if there is any unnecessary information.
- ❖ A sketch of the problem often helps their understanding.

STEP 2: STUDENTS SHOULD DECIDE ON A STRATEGY OR PLAN

Students should decide how they will solve the problem by thinking about the different strategies that could be used. They could try to make predictions, or guesses, about the problem. Often these guesses result in generalisations which help to solve problems. Students should be discouraged from making wild guesses but they should be encouraged to take risks. They should always think in terms of how this problem relates to other problems that they have solved. They should keep a record of the strategies they have tried so that they don't repeat them.

Some possible strategies include:

- ❖ Drawing a sketch, graph or table.
- ❖ Acting out situations, or using concrete materials.
- ❖ Organising a list.
- ❖ Identifying a pattern and extending it.
- ❖ Guessing and checking.
- ❖ Working backwards.
- ❖ Using simpler numbers to solve the problem, then applying the same methodology to the real problem.
- ❖ Writing a number sentence.
- ❖ Using logic and clues.
- ❖ Breaking the problem into smaller parts.

STEP 3: SOLVING THE PROBLEM

- ❖ Students should write down their ideas as they work so they don't forget how they approached the problem.
- ❖ Their approach should be systematic.
- ❖ If stuck, students should reread the problem and rethink their strategies.
- ❖ Students should be given the opportunity to orally demonstrate or explain how they reached an answer.

STEP 4: REFLECT

- ❖ Students should consider if their answer makes sense and if it has answered what was asked.
- ❖ Students should draw and write down their thinking processes, estimations and approach, as this gives them time to reflect on their practices. When they have an answer they should explain the process to someone else.
- ❖ Students should ask themselves 'what if' to link this problem to another. This will take their exploration to a deeper level and encourage their use of logical thought processes.
- ❖ Students should consider if it is possible to do the problem in a simpler way.

Teaching Notes Creating a Tree Diagram



A tree diagram can be used to represent the relationships between different factors in a problem. This strategy is most often used with problems where it is necessary to work out all possible combinations of the different factors in the problem. For example, if you have three different coloured wools to make a striped jumper, how many different ways can you organise the coloured stripes?

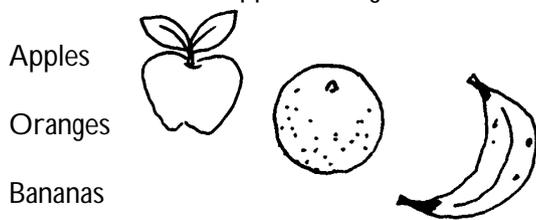
You must begin by making a list of the different colours, then work methodically down the list linking each different factor until you have covered all possible combinations.

Using a tree diagram enables the problem solver to visualise the different factors of the problem more easily. It ensures that a systematic approach is used and that no factors are missed out or repeated.

In order to use the strategy of drawing a tree diagram effectively, students will need to develop the following skills and understanding.

LISTING ALL THE NAMES OR OBJECTS

The first step in constructing a tree diagram is for students to identify all relevant factors in the problem and list them. The factors can be listed in any order as this does not affect the solution to the problem. For example, a class of students were asked to list their favourite fruits. In the list the students included apples, oranges and bananas.

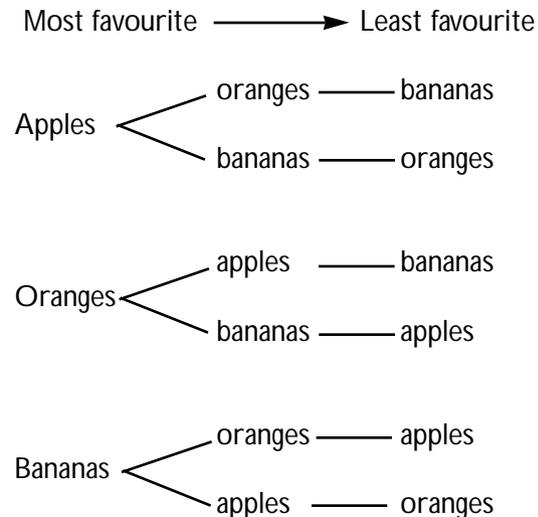


PAIRING OR CONNECTING INFORMATION USING LINES OR BRACKETS

Once the initial list had been compiled students were asked to organise the three types of fruit in order, from their least favourite to their most favourite. Every possible combination was covered by the students in the class. How many combinations were there?

Build on the initial list of apples, oranges and bananas by drawing two lines linking each fruit to the other two types of fruit. Then draw a line

joining each piece of fruit in this second list to whichever fruit hasn't already been linked to it. By doing this you will have a tree diagram showing every possible order.



There are six different orders in which the fruit could be listed.

WORKING METHODICALLY

By using a tree diagram and working through all possible combinations, a methodical approach to the problem is possible. Students should be encouraged to read through their tree diagram to check that each item has been linked to every other item in the problem and that no items have been omitted. It will be helpful for students to understand and remember this systematic approach so they can apply it to other similar problems.



Teaching Examples Creating a Tree Diagram



EXAMPLE 1

There are eight netball teams representing schools in the local area. The teams are playing in a netball competition to decide on the district champion. It is a knock-out competition, which means that once a team is beaten they leave the competition. How many games will be played during the entire competition?

Understanding the problem

WHAT DO WE KNOW?

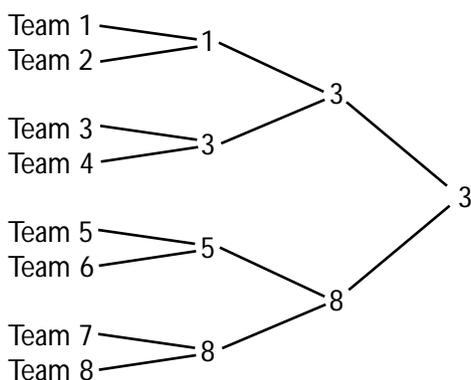
There are eight netball teams.
This is a knock-out competition.

WHAT DO WE NEED TO FIND OUT?

Questioning:
How many games will be played during the competition?

Planning and communicating a solution

Create your own knock-out competition by organising the eight teams to play one another. Begin by writing a list containing every team in the competition. Then pair each team with another team to show the first game of tennis. In the second line of the diagram write the names of the winners so that you only have half the number of teams (you can choose who will be the winners). Continue constructing your tree diagram in the same fashion until you only have one team left.



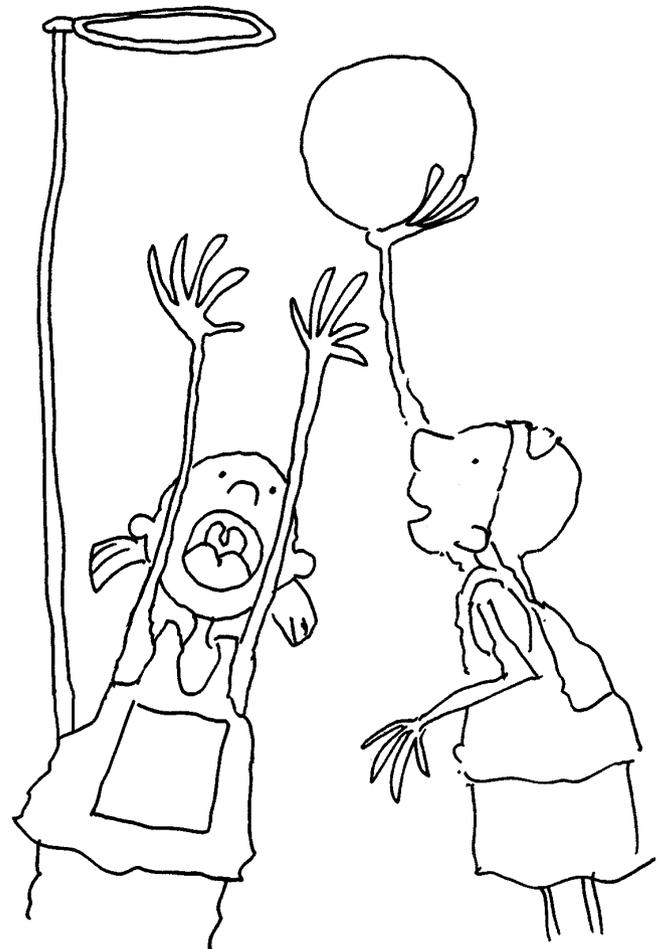
Seven games of tennis will be played during the competition.

Reflecting and generalising

By listing the teams and working methodically through each stage of the netball competition, it was easy to count how many games of netball were played. With problems of this type it is easy to miss out a team, or a stage in the competition, but by creating a tree diagram it was possible to visualise the problem and make sure no stages were missed.

Extension

This strategy can be applied to problems involving much higher numbers, or where teams are not knocked out, but have to play every other team. Students may wish to create similar problems themselves to practice using the tree diagram strategy.



Teaching Examples Creating a Tree Diagram



EXAMPLE 2

Lilly's dad is trying to decide what shoes and socks to wear on the first day of his new job. He has a pair of black shoes and a pair of brown shoes. He has five pairs of socks – blue, black, brown, green and grey. Lilly's dad wants to wear matching socks – and matching shoes. How many different combinations of shoes and socks are possible?

Understanding the problem

WHAT DO WE KNOW?

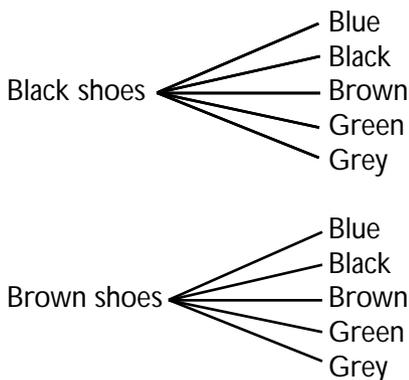
He has two pairs of shoes and five pairs of socks. He can wear any combination of shoes and socks.

WHAT DO WE NEED TO FIND OUT?

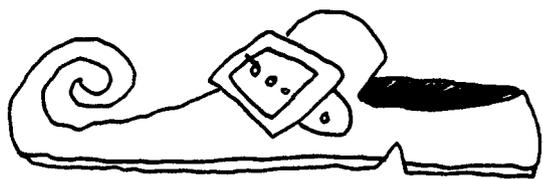
Questioning:
How many different combinations are there?

Planning and communicating a solution

Begin by listing the shoes, then use a tree diagram to connect each different pair of socks to both types of shoes.



There are five possible combinations with the black shoes and five possible combinations with the brown shoes, so there are ten combinations in all.

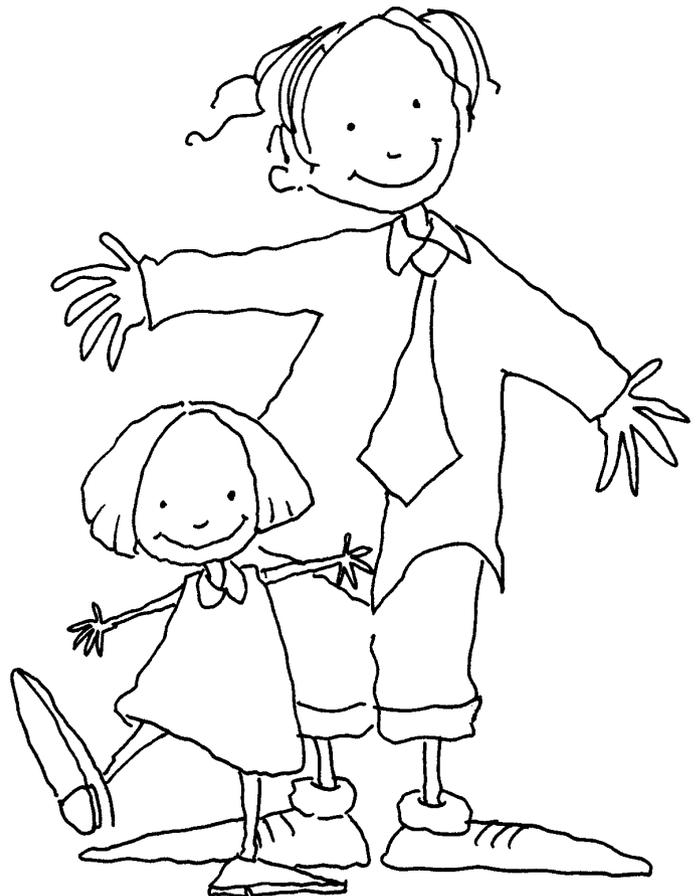


Reflecting and generalising

Even though this problem is quite straightforward, if you tried to solve it in your head, the problem would be much more difficult because it would be hard to remember whether you had counted every different pair of socks. Drawing the tree diagram makes the problem clearer because it provides a visual record of every element in the problem and the connections between them.

Extension

You can extend this problem by adding more pairs of shoes and socks, or by including different items of clothing such as trousers, shirts and ties. For example, how many different combinations are there if Lilly's dad has two pairs of trousers to match with his two pairs of shoes and five pairs of socks?



Teaching Examples Creating a Tree Diagram



EXAMPLE 3

Kim has four close friends – Karina, Rachel, Janine and Tricia. She is going to the pictures on Saturday but her mother says she can only invite two of her friends. How many choices does Kim have?

Understanding the problem

WHAT DO WE KNOW?

Kim has four friends.
She is only allowed to invite two of them to the pictures.

WHAT DO WE NEED TO FIND OUT?

Questioning:
How many different choices does she have?

Planning and communicating a solution

Begin your tree diagram by linking the first friend (Karina) to the other three friends. Then link the second friend to the two friends she hasn't already been linked to. Finally link the third friend to the one friend she hasn't already been linked to. Make sure you don't repeat any links. Note that the combination Karina and Rachel is the same as Rachel and Karina, so you don't need to repeat it.

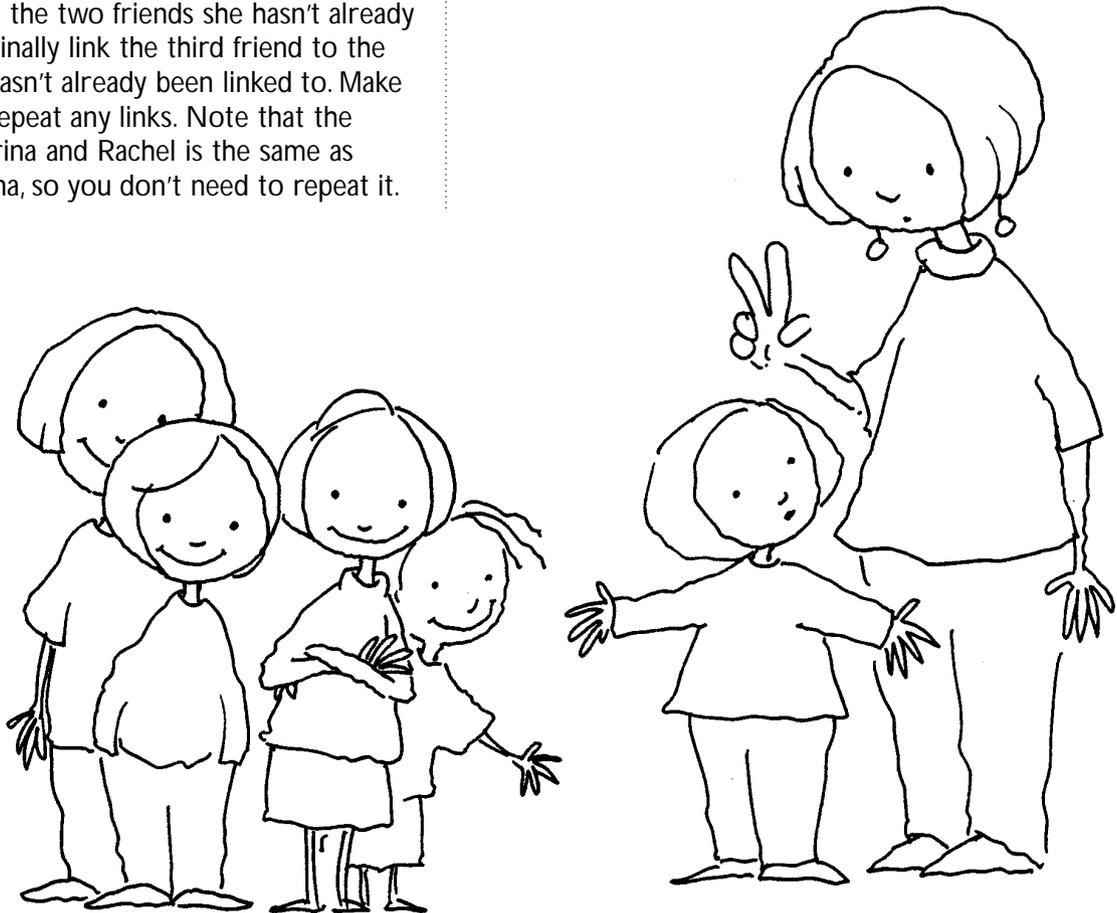
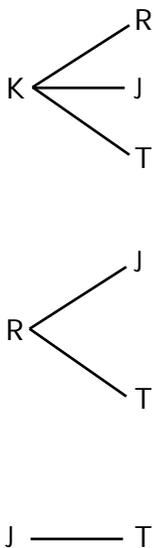
There are six possible choices Kim could make when deciding which friends to take to the pictures.

Reflecting and generalising

Creating a tree diagram enabled us to make sure that each factor was included in the diagram and that no one was left out. It was then easy to count all the friends in the diagram and reach a solution. The visualisation of the information assists with the calculation.

Extension

You can extend this problem by adding more friends for Kim to choose from or by allowing her to invite more people to the pictures. A slightly different problem would be to allow Kim to invite all her friends, but also to let her friends invite a number of their own friends. Then ask students how many people altogether would be going to pictures.



BLM Creating a Tree Diagram



★ Understanding the problem

What do you know? List the important facts from the problem.

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What do you need to find out?

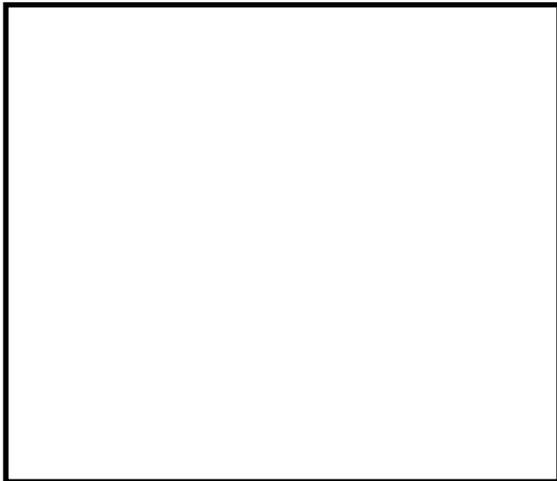
What is the problem asking you to do? What are you uncertain about?

Do you understand all aspects of the problem? Is there any unfamiliar or unclear language?

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★ Planning and communicating a solution

List the factors in the problem that make a starting point for your tree diagram.

Which factors must be connected to each other? Do you know how many levels will be in the tree diagram? Do you need to count all of the connections or only specific ones relevant to the question?

Have you checked to see that all information has been included?

★ Reflecting and generalising

Did the strategy work as planned? Is your answer correct? Will you be able to apply this method of problem solving to other similar problems? Could you have used a different method to solve the problem?

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★ Extension

How can this strategy be applied to more complicated problems involving additional factors?

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PROBLEM SOLVING TASK CARDS - Creating a Tree Diagram

Problem 1

Number **123**

Level
1

Ann is organising a card party and invites two of her friends to play. They each invite two other children. How many people will be playing cards?

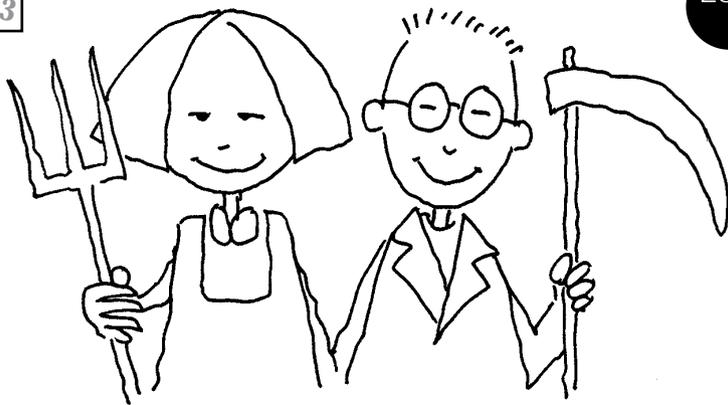


Problem 2

Number **123**

Level
1

Mr and Mrs Ling have four children. Each of their four children has two children of their own. How many grandchildren do the Lings have?

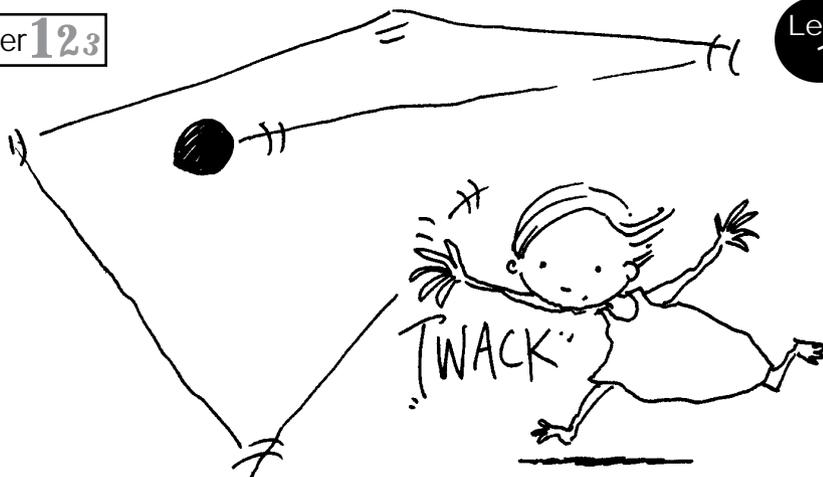


Problem 3

Number **123**

Level
1

At lunchtime Sita organises a knockout handball competition with three of her friends, Julie, Penny and Wei. How many games are played in the competition?



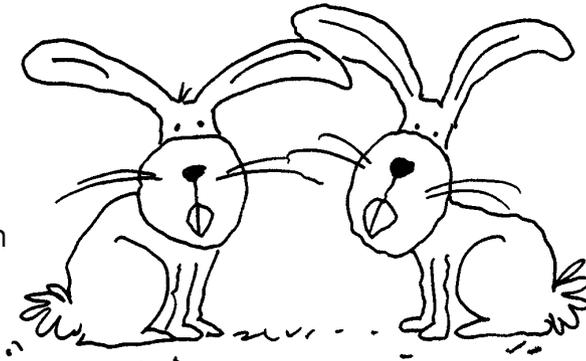
PROBLEM SOLVING TASK CARDS - Creating a Tree Diagram

Problem 4

Number **123**

Level
1

David had two rabbits, a male and a female. David's rabbits then had four female baby rabbits. Each of these babies grew up and gave birth to five rabbits. How many rabbits did David have in the end?

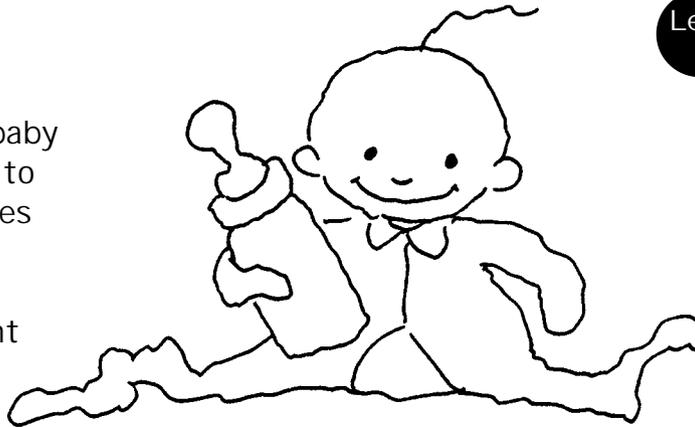


Problem 5

Number **123**

Level
1

Christina and Alex have a new baby boy but they can't decide what to name him. There are three names they like, Joe, Paul and George, but they only want to give the baby two names. What different choices do they have?

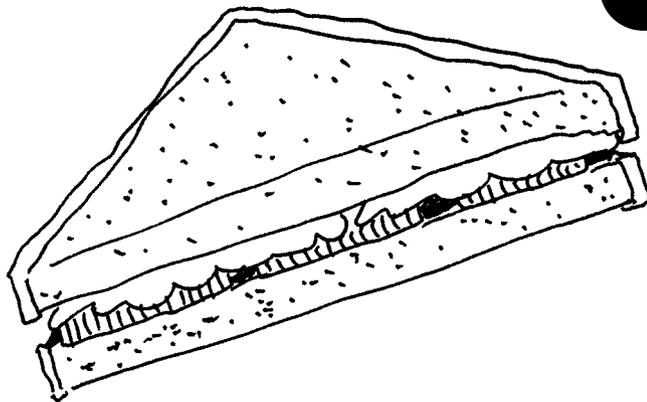


Problem 6

Number **123**

Level
1

Angela's father is making her a sandwich to take to school for lunch. She is allowed to have two things on her sandwich and she can choose from cheese, tomato, lettuce and ham. How many choices does she have?



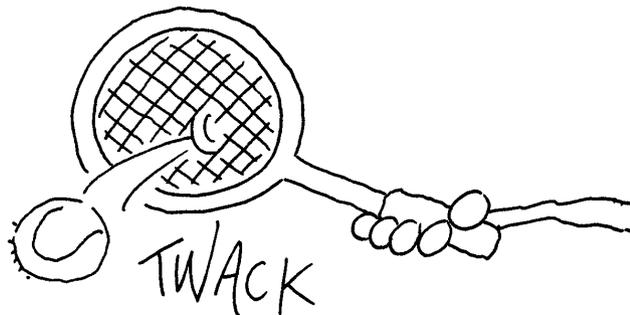
PROBLEM SOLVING TASK CARDS - Creating a Tree Diagram

Problem 7

Number **123**

Level
2

There are eight players in a tennis tournament. In every round the winners of each tennis game progress to the next round, but the loser is knocked out of the tournament. This continues until there is an overall winner of the tournament. How many games must be played to find the overall winner?

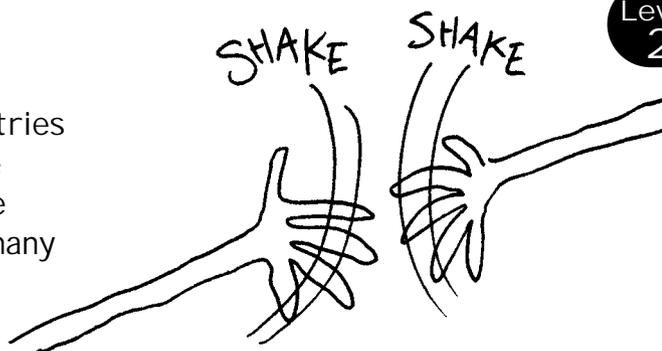


Problem 8

Number **123**

Level
2

Five students from different countries meet to plan an international peace ceremony. Each student shakes the hand of each other student. How many handshakes are there altogether?

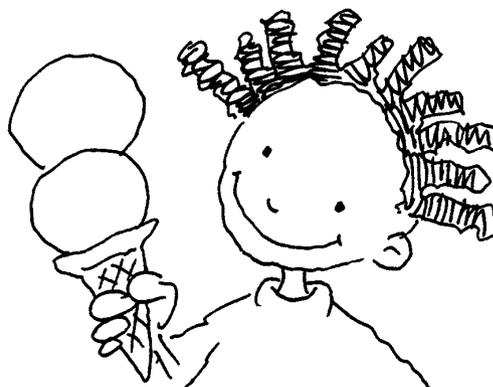


Problem 9

Number **123**

Level
2

Maria is buying an icecream cone. There are five different flavours of icecream for her to choose from: chocolate, strawberry, banana, mango and hazelnut, but she can only fit two flavours on her cone. What different choices does she have?



PROBLEM SOLVING TASK CARDS - Creating a Tree Diagram

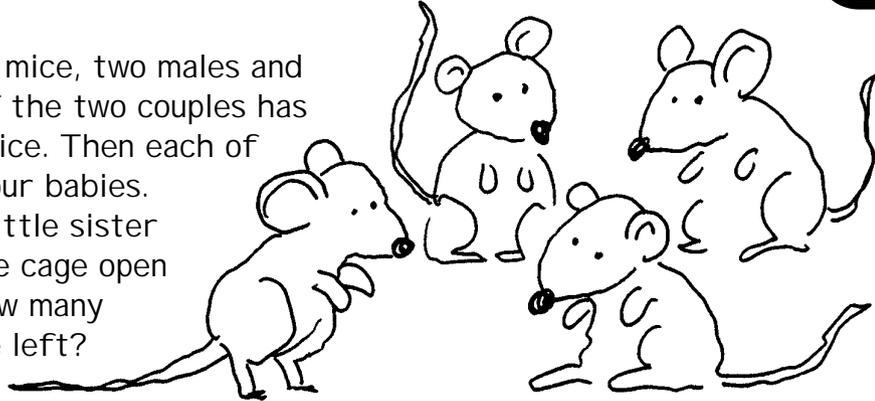
Problem 10

Number **123**

Level
2

Jaani has four white mice, two males and two females. Each of the two couples has three female baby mice. Then each of these females has four babies.

One night Jaani's little sister Aisha leaves the mice cage open and eight escape. How many mice does Jaani have left?

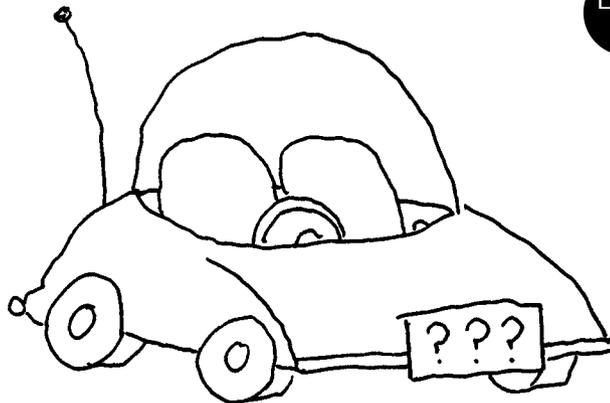


Problem 11

Number **123**

Level
2

There are four letters available to make up a car number plate, W, X, Y and Z. The number plate must have only three letters, but they can be placed in any order. How many different number plates can you make?

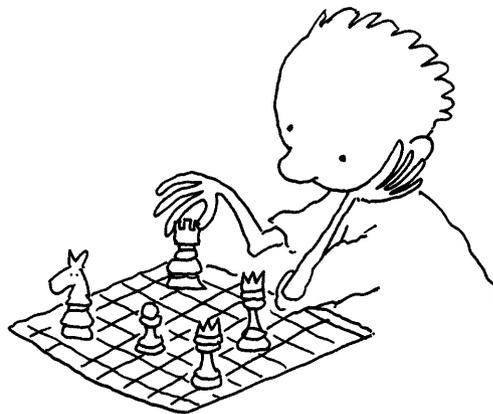


Problem 12

Number **123**

Level
2

The local youth club is organising a chess competition. Each player must play each other player in the competition. How many games will there be for the whole competition if there are six chess players?



PROBLEM SOLVING TASK CARDS - Creating a Tree Diagram

Problem 13 Number **123**

Level
3

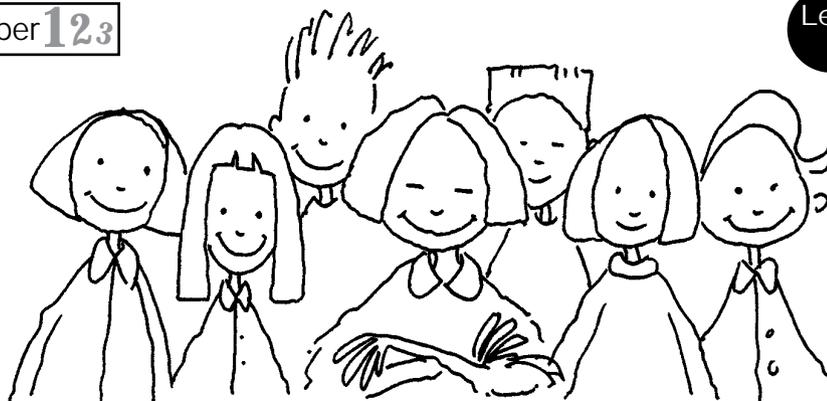
Lucy has four dolls in different national costumes, which she wants to arrange on a shelf in her bedroom. She has a doll from Spain, a doll from China, one from Indonesia and one from Mexico. Help Lucy arrange her dolls by working out all the different ways she can arrange them. How many different ways are there?



Problem 14 Number **123**

Level
3

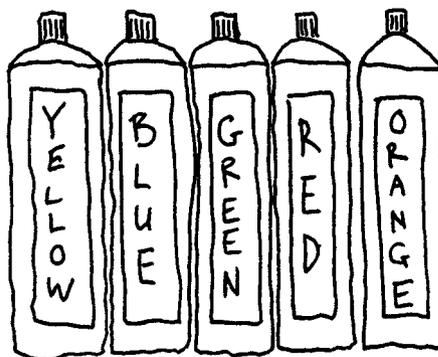
A chairman and vice-chairman are to be elected from seven candidates. How many different results are possible?



Problem 15 Number **123**

Level
3

A class enters a competition to design a new flag. They are given a choice of five colours: blue, red, green, orange and yellow. The flag must have three bands of colour, but the colours can be in any order. How many different flags can the class design?



PROBLEM SOLVING TASK CARDS - Creating a Tree Diagram

Problem 16 Number **123**

Level
3

Sally has a bag of mixed lollies. The bag contains toffees, mints, fruit drops, smarties, jelly beans, candies and licorice. Sally allows everyone in her class to choose three lollies, but they must not choose more than one of each kind. How many different combinations of lollies are possible?



Problem 17 Number **123**

Level
3

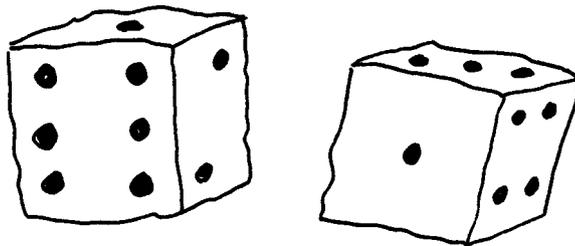
Mr Johnson wants to wear a three piece outfit to his birthday party. In the wardrobe he has a white shirt and a pale blue shirt. He has a pair of brown trousers, a pair of black trousers and some blue jeans. He has a cashmere coat and a windcheater. How many different three piece outfits can he wear?



Problem 18 Number **123**

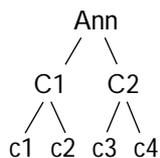
Level
3

If you have two dice and you throw them once, what possible combinations of numbers can you get?



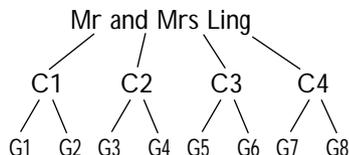
Answers to Task Cards

Problem 1



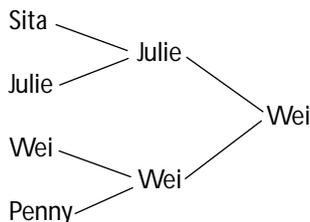
There will be seven people playing cards.

Problem 2



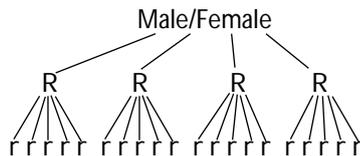
The Lings have eight grandchildren.

Problem 3



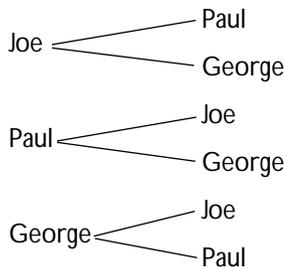
There are three games in the competition.

Problem 4



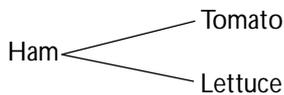
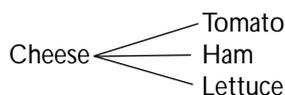
David has 26 rabbits in the end.

Problem 5



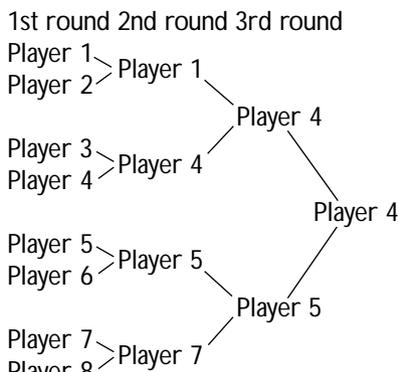
There are six choices.

Problem 6



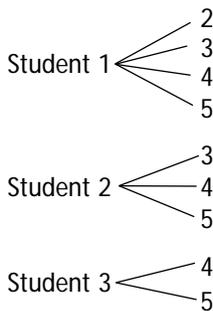
Tomato — Lettuce
Angela has six choices of different sandwiches.

Problem 7



There are seven games of tennis played altogether.

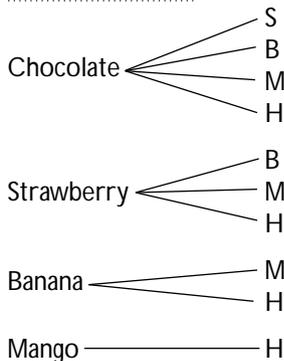
Problem 8



Student 4 — 5

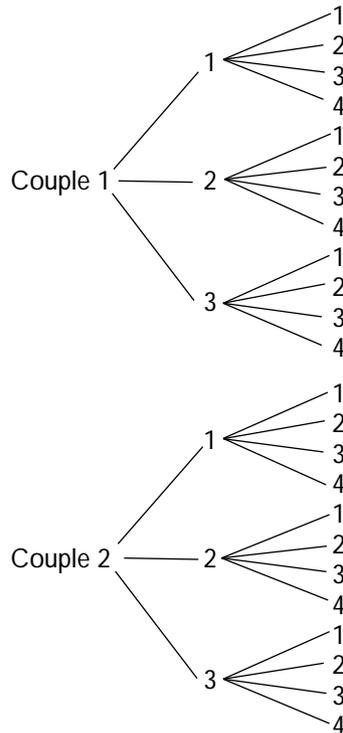
There are ten handshakes.

Problem 9



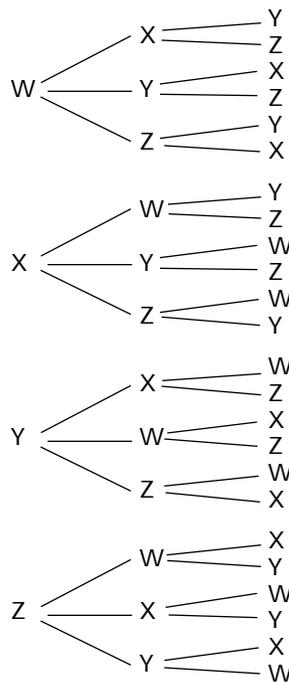
There are ten different combinations.

Problem 10



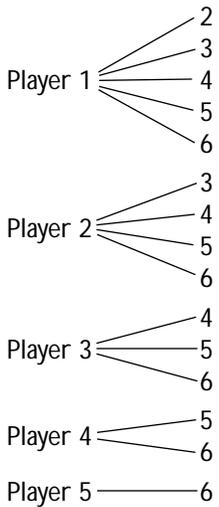
There are 34 mice, but then eight escape so, $34 - 8 = 26$. Jaani is left with 26 mice.

Problem 11



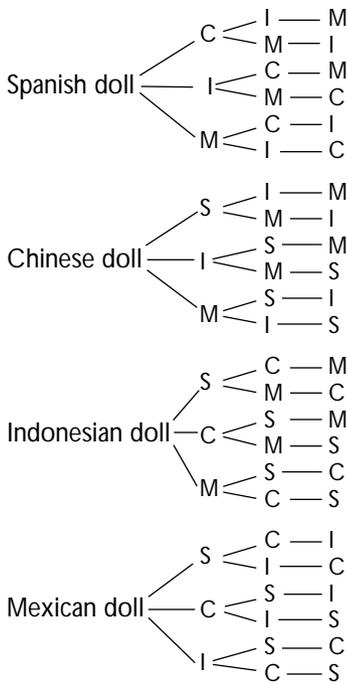
There are 24 different number plates.

Problem 12



There will be 15 games for the whole chess competition.

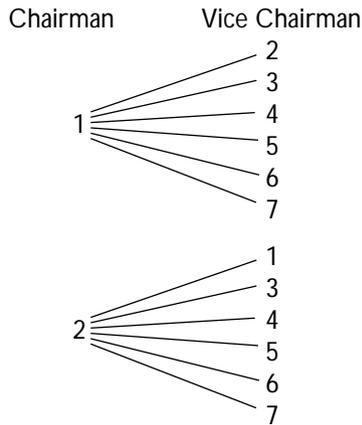
Problem 13



There are 24 different ways Lucy can organise her dolls.

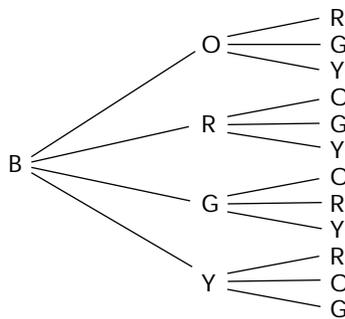


Problem 14



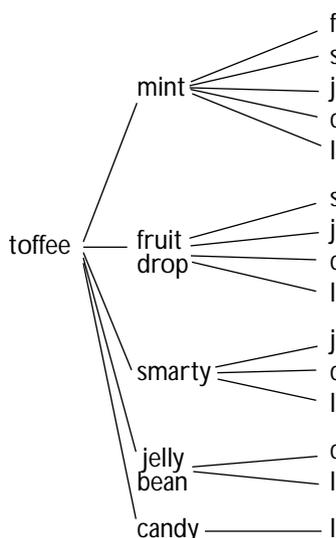
There are six combinations for each candidate, so for the seven candidates there are 42 combinations in all.

Problem 15



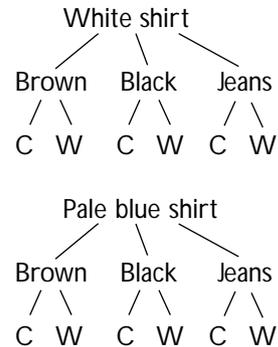
$12 \times 5 = 60$, so there are 60 possible flags the students can design.

Problem 16



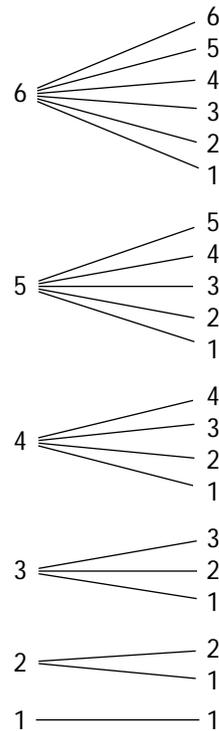
There are 15 different combinations.

Problem 17



Mr Johnson had 12 possible three piece outfits.

Problem 18



There are 21 different combinations.

