UNIT P1
Problem Solving
Middle Primary


Blake's Topic Bank

# Problem Solving
## Drawing a Diagram

by Sharon Shapiro

This unit contains:

- Teaching notes
- 3 teaching examples
- 1 BLM
- 18 task cards
- Answers

# Problem Solving
## Drawing a Diagram Middle Primary

***Sharon Shapiro***

## The Problem Solving Process

*It is important that students follow a logical and systematic approach to their problem solving. Following these four steps will enable students to tackle problems in a structured and meaningful way.*

### Step 1: Understanding the problem

- Encourage students to read the problem carefully a number of times until they fully understand what is wanted. They may need to discuss the problem with someone else or rewrite it in their own words.
- Students should ask internal questions such as, what is the problem asking me to do, what information is relevant and necessary for solving the problem.
- They should underline any unfamiliar words and find out their meanings.
- They should select the information they know and decide what is unknown or needs to be discovered. They should see if there is any unnecessary information.
- A sketch of the problem often helps their understanding.

### Step 2: Students should decide on a strategy or plan

Students should decide how they will solve the problem by thinking about the different strategies that could be used. They could try to make predictions, or guesses, about the problem. Often these guesses result in generalisations which help to solve problems. Students should be discouraged from making wild guesses but they should be encouraged to take risks. They should always think in terms of how this problem relates to other problems that they have solved. They should keep a record of the strategies they have tried so that they don't repeat them.

Some possible strategies include:

- Drawing a sketch, graph or table.
- Acting out situations, or using concrete materials.
- Organising a list.
- Identifying a pattern and extending it.
- Guessing and checking.
- Working backwards.
- Using simpler numbers to solve the problem, then applying the same methodology to the real problem.
- Writing a number sentence.
- Using logic and clues.
- Breaking the problem into smaller parts.

### Step 3: Solving the problem

- Students should write down their ideas as they work so they don't forget how they approached the problem.
- Their approach should be systematic.
- If stuck, students should reread the problem and rethink their strategies.
- Students should be given the opportunity to orally demonstrate or explain how they reached an answer.

### Step 4: Reflect

- Students should consider if their answer makes sense and if it has answered what was asked.
- Students should draw and write down their thinking processes, estimations and approach, as this gives them time to reflect on their practices. When they have an answer they should explain the process to someone else.
- Students should ask themselves 'what if' to link this problem to another. This will take their exploration to a deeper level and encourage their use of logical thought processes.
- Students should consider if it is possible to do the problem in a simpler way.

# Teaching Notes Drawing a Diagram

Drawing a picture of a word problem often reveals aspects of the problem that may not be apparent at first. If the situation described in the problem is difficult to visualise, a diagram, using simple symbols or pictures, may enable students to see the situation more easily. The diagram will also assist students to keep track of the stages of a problem where there are a number of steps.

In order to use the strategy of drawing a diagram effectively, students will need to develop the following skills and understanding.

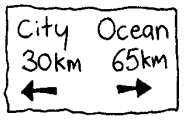
## USING A LINE TO SYMBOLISE AN OBJECT

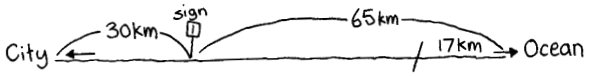
Simple line drawings help students to visualise a situation. In a problem students were asked how many markers would be needed if they placed a marker at every two metre point on a ten metre rope. In response, students may calculate mentally $10 \div 2 = 5$, so five posts are needed. However, if students draw the rope and markers, they will see that actually six markers are needed because we need one for the starting point and another at the end of the rope.

## USING A TIME/DISTANCE LINE TO DISPLAY THE INFORMATION

A time/distance line helps to show distance, or movement from one point to another. Students were asked to calculate how far they are from the city when they are 17 kilometres from the ocean, using the information on this signpost.

Students should draw a line and on it write the distances.




30 km + (65 – 17 km) = 78 km

## SCALE

When students are required to draw a diagram of a large area the diagram will often need to be scaled down.

For example, in a drawing, one centimetre could have the value of one kilometre.

Alternatively, a one centimetre line could represent ten kilometres or even 500 kilometres, depending on the scale of the drawing.

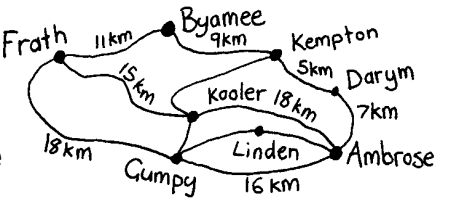
Show students how to use scaled down measurements to solve a problem, then convert the solution to the actual measurements.

## MAPPING OR SHOWING DIRECTION

Students will often be faced with drawings that require them to have an understanding of direction. They will also meet problems where they are asked to plot a course by moving up, down, right or left on a grid. They will also meet to use the compass points to direct themselves — north, south, east, west, north-easterly, south-westerly, and so on.

They will also need to become familiar with measurement words which may be unfamiliar to them, such as pace. Opportunities should be given for the students to work out how many paces it takes to cover the length and breadth of the classroom or to pace out the playground, so they develop a means of comparison.

Students should develop the ability to use a map as the focus of the problem they are solving.

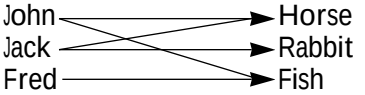



They should be able to plot four different routes from Byamee to Gumpy without passing through any town twice.

## SHOWING THE RELATIONSHIPS BETWEEN THINGS

Students will find it helpful to draw diagrams and use symbols in order to visualise the relationships between things.

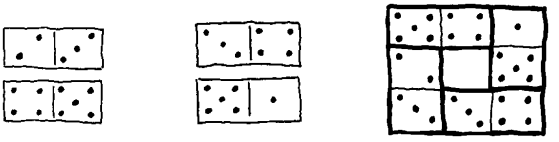
For example:




## DRAWING A PICTURE

Drawing a picture can help students organise their thoughts and so simplify a problem.

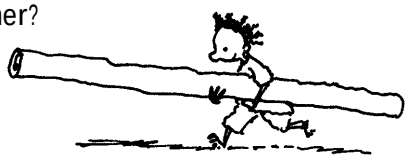
These four domino pieces have to be organised in a square shape with each side of the square adding up to a total of ten.



# Teaching Examples Drawing a Diagram

## EXAMPLE 1

The children built a log playhouse in a square shape. They used eight vertical posts on each side of the house. How many posts did they use altogether?



### Understanding the problem

#### WHAT DO WE KNOW?

We know the playhouse was a square.
We know how many posts were on each side.

#### WHAT DO WE NEED TO FIND OUT?

Questioning:
Do we understand the meaning of vertical posts?
How many posts were used altogether?

### Planning and communicating a solution

#### WHAT WE DID

It is important that students develop their abilities to logically explain their strategy. They should use some mathematical language and drawings such as graphs, charts and diagrams in their explanation or during the problem solving process.

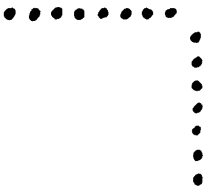
Students may suggest that the solution is easy because there are four sides and so eight posts times four sides will equal 32 posts. This is incorrect. Students should be encouraged to draw a picture so that the problem is clarified. They will then be able to see and count the posts.

### Step-by-step explanation

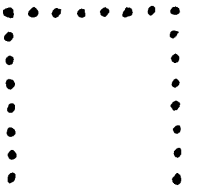
Here is a step-by-step explanation of the process.

Let ▪ stand for one post. Ask students to draw one side of the playhouse first.

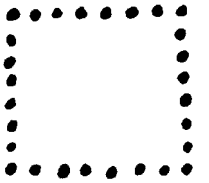
They should then continue with the second side. This is a perfect time to discuss whether the corner posts are used once or twice on each side.



Students will see that, because the same corner post is used for both the horizontal and vertical sides, only seven posts are added. The third line will again reuse the corner post so only seven posts are added.



When students draw the final side it is important that they count the posts carefully to ensure that only eight posts are used. Because they are reusing both corner posts only six others need to be added.



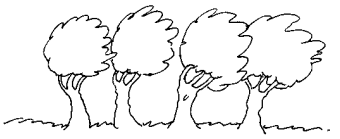
When the posts are carefully counted we see that 28 posts were used.

### Reflecting and generalising

Once students have reflected on the solution, they can generalise about problems of this type and see how this solution can be applied to similar problems. They should consider the method used to see if it can be improved. They should think about the accuracy of the method. They should question if there is a shorter or different method they could have chosen? Suitable technology such as blocks can be used to assist with the investigation by replacing pen and paper or can be used to double-check the answer.

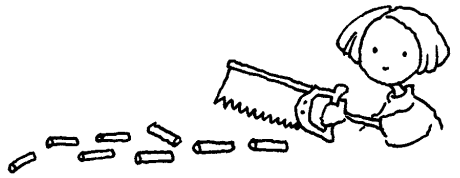
### Extension

What if the square playhouse was a rectangle, how would this affect the answer? What if 12 vertical posts were used? What if the shape was a rectangle but 26 posts were used and the length and breadth weren't specified? How many different ways could the posts be placed? What if there were 36 posts altogether?



## EXAMPLE 2

A thick dowel stick has to be cut into eight pieces. Each cut takes you 30 seconds. How long will it take to cut the dowel stick into pieces?



### Understanding the problem

#### WHAT DO WE KNOW?

We know we need eight pieces of dowel.
We know each cut takes 30 seconds.

#### WHAT DO WE NEED TO FIND OUT?

Questioning:
How many cuts will we make? How long will the cuts take? Is this simply a numerical problem or must a diagram be drawn?

### Planning and communicating a solution

#### WHAT WE DID

A line was drawn to symbolise the stick.

In order to make the eight pieces we cut the dowel in seven places.

—|—|—|—|—|—|—|—

Seven cuts multiplied by 30 seconds per cut equals, 7 x 30 = 210 seconds. It will take 210 seconds or 3 minutes and 30 seconds to make the cuts.

### Reflecting and generalising

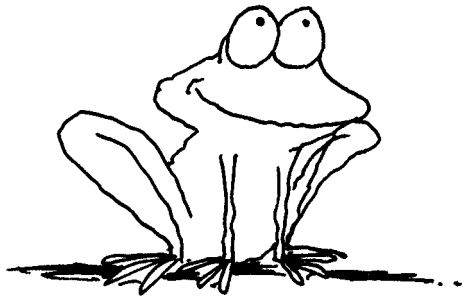
Students who simply multiplied the eight pieces by 30 seconds were working inaccurately and had not visualised the problem. They needed a concrete item such as a strip of paper or piece of plasticine that can be cut into pieces. We are able to generalise that in future if a larger piece is cut into smaller pieces, the end pieces are not cut and a drawing will clarify this idea.

### Extension

What if the students were building a cubby house and it took three minutes to join each plank securely to the others. How long would it take them to build the walls if they have six vertical planks on each side?

## EXAMPLE 3

A frog fell down an abandoned well which was 21 metres deep. He found it difficult to jump up the mud coated walls. He started his long jump up the well at six a.m. It took him 15 minutes to jump three metres because the walls were so slippery. At the end of every 15 minute period he rested for five minutes while he sadly slipped down one metre. He continued on at the same rate. At what time did he finally reach the top of the well?



### Understanding the problem

#### WHAT DO WE KNOW?

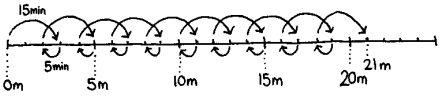
The well was 21 metres deep.
The frog started jumping at six a.m.
He jumped three metres in fifteen minutes.
He slipped down one metre at the end of every fifteen minute period.

#### WHAT DO WE NEED TO FIND OUT?

Questioning:
What time did the frog finally reach the top of the well?

### Planning and communicating a solution

This problem can be approached in a number of ways. Students can use 25 squares of one centimetre grid paper or they can draw a time line with one centimetre marks. One centimetre would represent one metre the frog jumps or slips. Drawing the line will help students to visualise the problem.



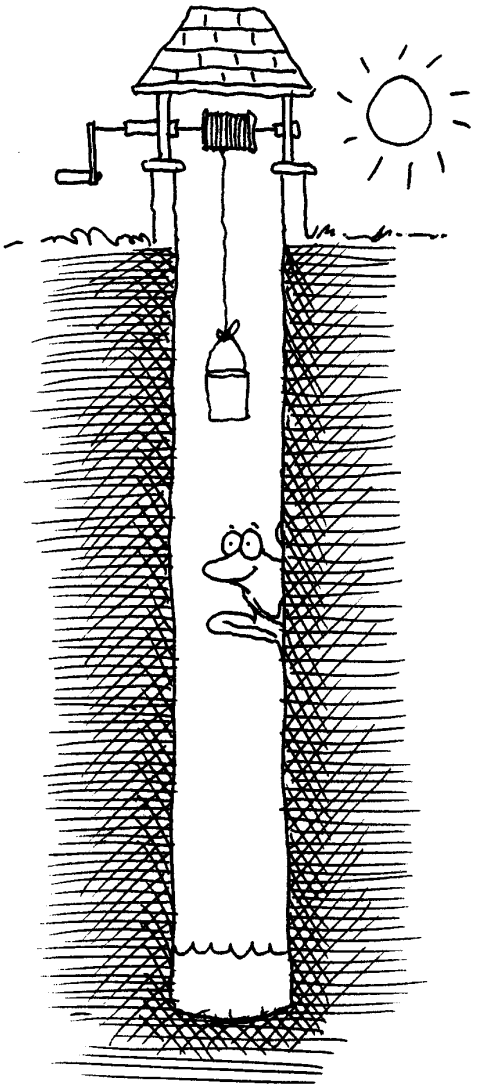

It would take the frog 195 minutes to cover the distance. He would reach the top of the well at 9:15 a.m.

### Reflecting and generalising

It is important that students realise that there is more than one way to approach a problem. The time line or grid paper will assist them to visualise the problem but an alternative way to solve the problem could have been drawing a table.

### Extension

What if it hailed for an hour and fifteen minutes and the frog was forced to retreat to the bottom of the well after covering eighteen metres. The frog would have to start its jump once more. How long would the trip take now?



# BLM Drawing a Diagram

## ★ Understanding the problem

List what you know from reading the problem.....

.....

.....

What do you need to find out?
What questions do you have?
What are you uncertain about?
Is there any unfamiliar or unclear language?

.....

.....

.....

.....

## ★ Planning and communicating a solution

Which solution will you try?
Will you draw a line to symbolise the objects? Draw a time line to show passing time or distance covered? Draw a picture to stand for objects? Trace a journey on a map? Use directions on a compass? Use a scale drawing? Show the relationship between things using a diagram or symbols?

## ★ Reflecting and generalising

What did you find?
How accurate is your answer? How can the strategy you used be applied to other situations? Could another more effective method have been used? Is there a shorter or different method?

.....

.....

.....

## ★ Extension

How can this problem be extended? What factors can be added as part of a 'what if' question?

.....

.....

.....

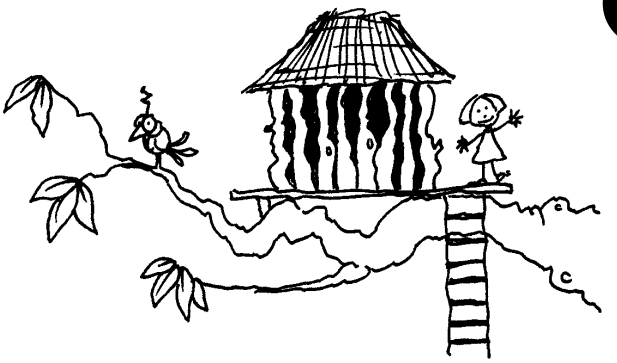
# PROBLEM SOLVING TASK CARDS - Drawing a Diagram

## Problem 1

Space

Level 1

Jane built a square cubby house on one side of the river. She used eight vertical tree branches for each side. How many branches did she use altogether?



## Problem 2

Space

Level 1

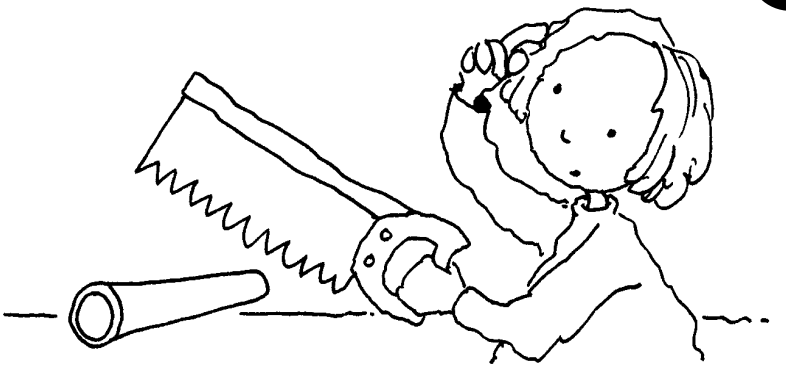
Brett built a tower using four different coloured milk cartons. The red carton was below the green carton. The blue carton was above the yellow carton which was above the green carton. Which carton is on top?


## Problem 3

Measurement

Level 1

Giovanni has to saw a pipe into seven pieces. If it takes him four minutes to make one cut, how long will it take to cut into seven pieces?



# PROBLEM SOLVING TASK CARDS - Drawing a Diagram

## Problem 4

Measurement

Level 1

A spider is climbing up a 30 metre building. Each day it climbs five metres and slides back one metre. How many days will it take to reach the top?

## Problem 5

Measurement

Level 1

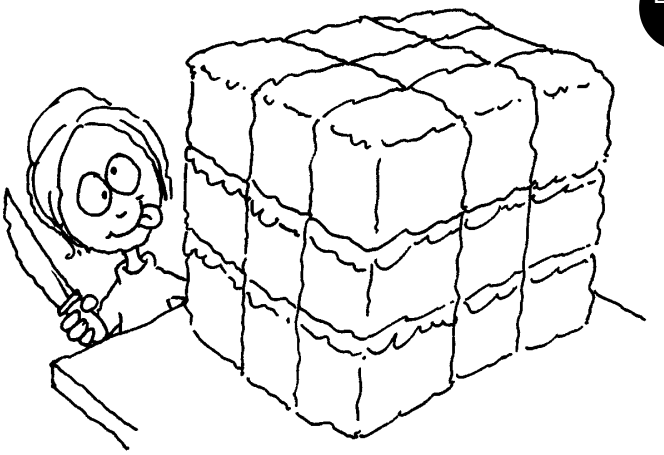
Jacob is building a Lego figure. It takes him one and a half seconds to join two pieces. How long will it take him to join nine pieces into one long strip?

## Problem 6

Space

Level 2

Jeremy's birthday cake was baked in the shape of a cube and was covered on every side with delicious pink icing. If it was cut into 27 cubes, how many pieces would have icing on no, one, two or three sides?



# PROBLEM SOLVING TASK CARDS - Drawing a Diagram

## Problem 7

Space

Level 2

In the Year Four classroom the desks are organised in equal rows. Jane sits in the desk that is fourth from the front and third from the back. There are four desks on the right but only one to the left of Jane's desk. How many desks are in the room?

## Problem 8

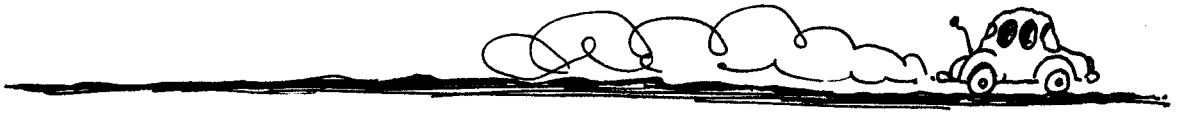
Space

Measurement

Level 2

Mrs Williams became terribly lost on the way to an important meeting. She stopped and asked a farmer the way. He told her that some of the roads were flooded and she would have to travel a long way round to get to her meeting. The farmer told Mrs Williams to drive for four kilometres north, then to continue on for five kilometres in a westerly direction. She was then to travel south for two kilometres and then turn to the east for one kilometre and finally to travel north for one kilometre. She would then arrive safely at her meeting.

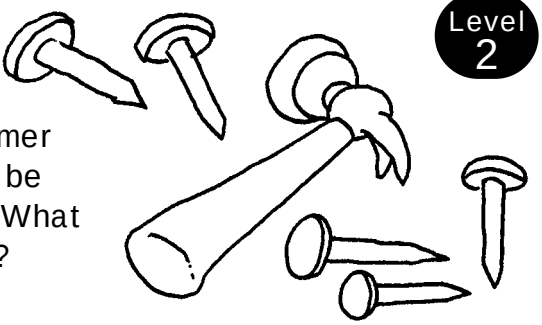
So she could remember the way, Mrs Williams drew a map on one centimetre grid paper. Help her draw the map on the grid paper.



## Problem 9

Measurement

Level 2



For her woodwork project, Angela has to hammer five nails into a piece of wood. The nails must be in a straight line and 0.75 centimetres apart. What is the distance from the first nail to the last?

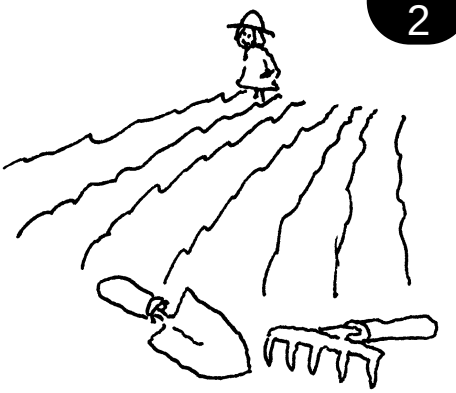
# PROBLEM SOLVING TASK CARDS - Drawing a Diagram

## Problem 10

Measurement

Level 2

Mrs Harriman decided to enclose an area of her garden to use as a vegetable patch. She had to make sure that the rectangular area was fenced on all sides so that the sheep would not wander in and munch on her vegetables. She used 26 posts to make the entire fence but used five more posts on the longer sides than the shorter sides. How many posts are on each side?



## Problem 11

Number 1 2 3

Level 2

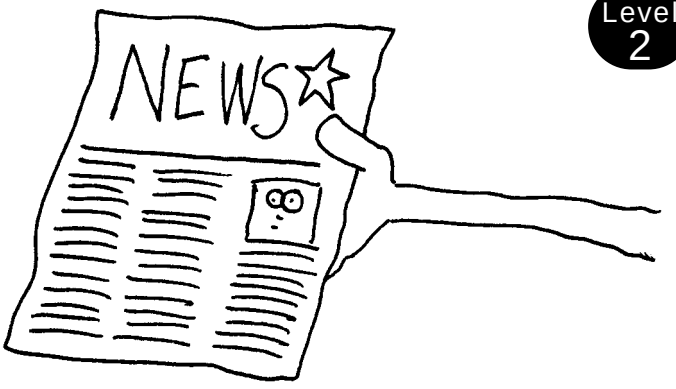
On a camp, the students are lining up to collect their breakfast. There are 50 students in front of Ned. Ned is hungry so he decides to move towards the front of the line. Each time one person is given their tray, Ned slips past two students. How many students will be given their breakfast before Ned?

## Problem 12

Number 1 2 3

Level 2

Two hundred and twenty seven students were standing at assembly. Every tenth student was chosen to collect a news sheet. How many were chosen?



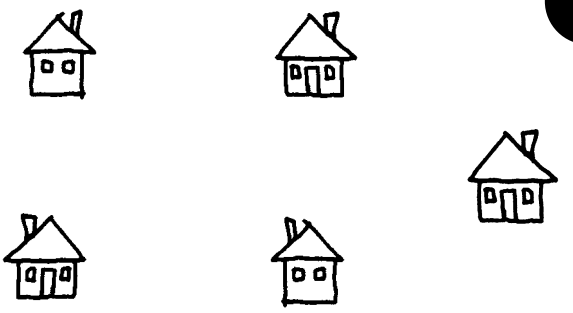
# PROBLEM SOLVING TASK CARDS - Drawing a Diagram

## Problem 13

Space

Level 3

Five families are building project homes in an isolated area. Roads will have to be built to connect each house with all the other houses. How many roads have to be built?



## Problem 14

Space

Level 3

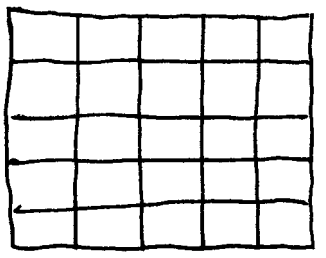
A gardener is asked to plant ten trees in five rows, with each row containing four trees. How did he do this?

## Problem 15

Space

Level 3

Ten objects must be buried in the garden for a treasure hunt. To make this easier the garden has been divided into an imaginary grid, five squares across by five squares down.



To make the treasure hard to find it is important to hide the objects in different areas. You must make sure that no more than two objects lie in a line in any direction.

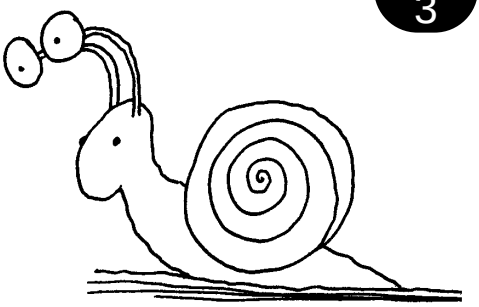
# PROBLEM SOLVING TASK CARDS - Drawing a Diagram

## Problem 16

Measurement

Level 3

A snail finds itself at the bottom of a deep well. The well is 1530 centimetres deep. Each day the snail struggles up 180 centimetres and then stops to rest. While it is resting the snail slides down 30 centimetres. How long before it reaches the top of the well?

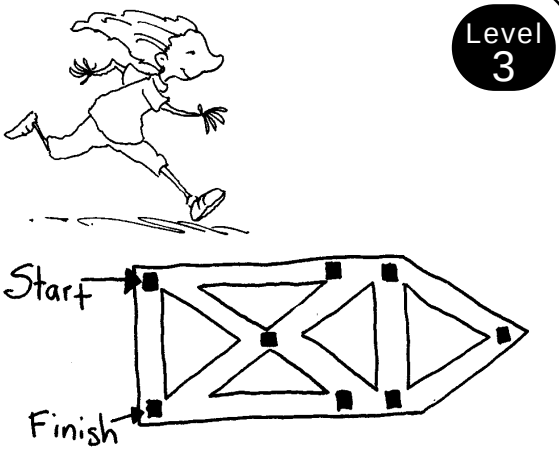


## Problem 17

Measurement

Level 3

The streets around the school have been closed off for the annual school marathon. During the marathon, the students must stay on the road and pass through all the checkpoints. Which is the quickest route they could choose without travelling through any part of the route more than once?

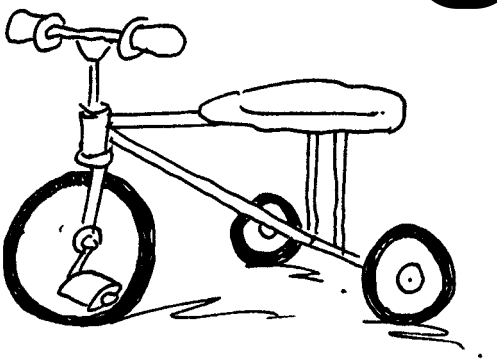



## Problem 18

Measurement

Level 3

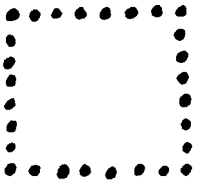
Eric's back garden is 10 metres by 14 metres in size. Each day Eric rides his tricycle down the three metre long path leading from the back door of his house to the garden and around the very edge of the garden four times. Then he rides back up the path where he leaves his tricycle. How far does he ride each day?



# Answers to Task Cards

## Problem 1

Jane used 28 tree branches to build her square cubby house.

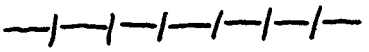


## Problem 2

The blue milk carton is at the top of Brett's tower.

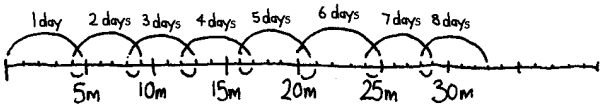

## Problem 3

Giovanni has to make six cuts which take him four minutes each, so 6 x 4 = 24 minutes.



## Problem 4

The spider takes eight days to reach the top.




## Problem 5

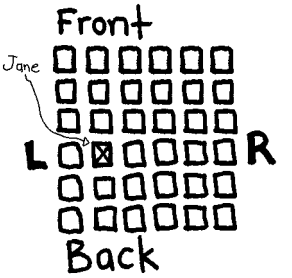
Jacob had to make eight joins to connect his nine pieces of Lego and each one took one and a half seconds, so 8 x $1\frac{1}{2}$ = 12 seconds

## Problem 6

| 0 pink sides | 1 pink side | 2 pink sides | 3 pink sides |
|---|---|---|---|
| 1 | 6 | 12 | 8 |

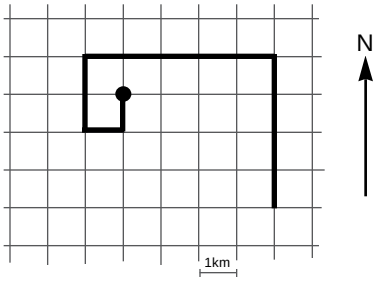
## Problem 7

There are six desks in each row from the front to the back of the room and six desks in each row across the room, so there are 36 desks in the room, 6 x 6 = 36.



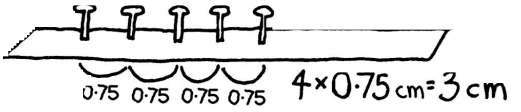

## Problem 8

Here is Mrs Williams' map of her route to the meeting.



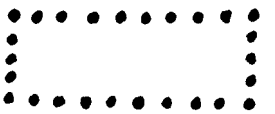

## Problem 9

Angela's line of nails is three centimetres from the first nail to the last.




## Problem 10

Mrs Harriman used ten posts on the long sides of her fence and five posts on the short sides.



## Problem 11

Either 16 or 17 students will be fed before Ned, depending on whether a person is served before Ned makes his move or whether Ned moves first before anyone has been served.

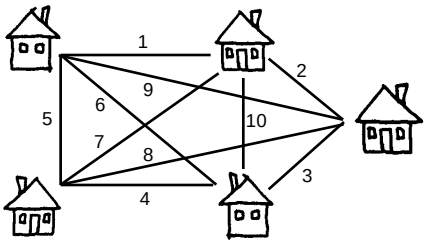
## Problem 12

22 students were chosen to collect a news sheet.

10 students in 100, 20 students in 200
2 students in 27
equals 22 students in total

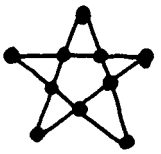
## Problem 13

Ten roads have to be built to connect each house with all the others.

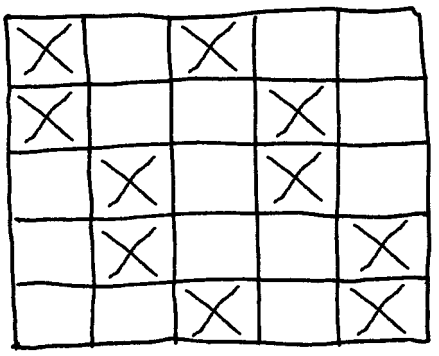



## Problem 14

Here is how the gardener planted his ten trees in five rows.



## Problem 15



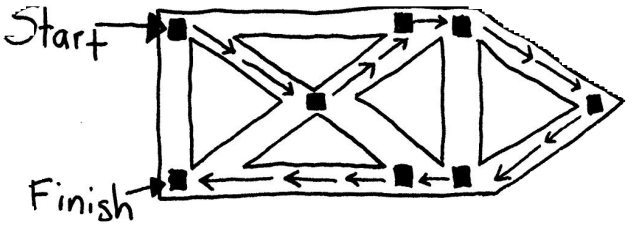
## Problem 16

The snail reached the top of the well on the tenth day.

| Day | Climbs to (cms) | Slides to (cms) |
|---|---|---|
| 1 | 180 | 150 |
| 2 | 330 | 300 |
| 3 | 480 | 450 |
| 4 | 630 | 600 |
| 5 | 780 | 750 |
| 6 | 930 | 900 |
| 7 | 1080 | 1050 |
| 8 | 1230 | 1200 |
| 9 | 1380 | 1350 |
| 10 | 1530 | |

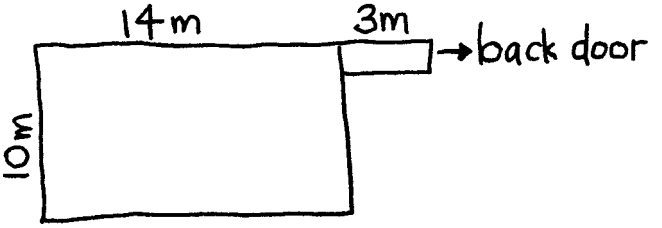
## Problem 17

The route shown on the diagram below is quickest.




## Problem 18

Eric rides 198 metres each day.




1 circuit around the garden, 10 m + 14 m + 10 m + 14 m = 48 m
4 circuits, 48 m x 4 = 192 m

Up and back along the driveway 2 x 3 m = 6 m

Total distance 192 m + 6 m = 198 metres