UNIT P12
Problem Solving
Upper Primary

Blake's Topic Bank

Problem Solving Using Logical Reasoning

by **Sharon Shapiro**

This unit contains:

- Teaching notes
- 3 teaching examples
- 1 BLM
- 18 task cards
- Answers

Problem Solving
Using Logical Reasoning

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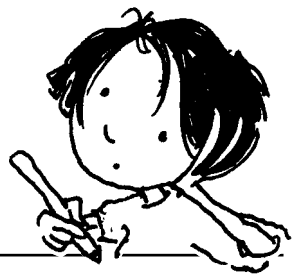
Upper Primary



The Problem Solving Process

It is important that students follow a logical and systematic approach to their problem solving. Following these four steps will enable students to tackle problems in a structured and meaningful way.

Step 1: Understanding the problem

- Encourage students to read the problem carefully a number of times until they fully understand what is wanted. They may need to discuss the problem with someone else or rewrite it in their own words.
- Students should ask internal questions such as, what is the problem asking me to do, what information is relevant and necessary for solving the problem.
- They should underline any unfamiliar words and find out their meanings.
- They should select the information they know and decide what is unknown or needs to be discovered. They should see if there is any unnecessary information.
- A sketch of the problem often helps their understanding.

Step 2: Students should decide on a strategy or plan

Students should decide how they will solve the problem by thinking about the different strategies that could be used. They could try to make predictions, or guesses, about the problem. Often these guesses result in generalisations which help to solve problems. Students should be discouraged from making wild guesses but they should be encouraged to take risks. They should always think in terms of how this problem relates to other problems that they have solved. They should keep a record of the strategies they have tried so that they don't repeat them.

Some possible strategies include:

- Drawing a sketch, graph or table.
- Acting out situations, or using concrete materials.
- Organising a list.
- Identifying a pattern and extending it.
- Guessing and checking.
- Working backwards.
- Using simpler numbers to solve the problem, then applying the same methodology to the real problem.
- Writing a number sentence.
- Using logic and clues.
- Breaking the problem into smaller parts.

Step 3: Solving the problem

- Students should write down their ideas as they work so they don't forget how they approached the problem.
- Their approach should be systematic.
- If stuck, students should reread the problem and rethink their strategies.
- Students should be given the opportunity to orally demonstrate or explain how they reached an answer.

Step 4: Reflect

- Students should consider if their answer makes sense and if it has answered what was asked.
- Students should draw and write down their thinking processes, estimations and approach, as this gives them time to reflect on their practices. When they have an answer they should explain the process to someone else.
- Students should ask themselves 'what if' to link this problem to another. This will take their exploration to a deeper level and encourage their use of logical thought processes.
- Students should consider if it is possible to do the problem in a simpler way.

Teaching Notes Using Logical Reasoning

Logical reasoning is unlike the 'guess and check' strategy, where random guesses are made. When logical reasoning is used, the problem solver begins with the knowledge that each piece of information is a piece of the puzzle, and that by putting the pieces together, they will be working towards a solution. The problem is then tackled step by step.

Students can approach these problems in different ways, working by a process of elimination, or using each piece of information to build towards the solution.

Read each clue thoroughly

Reading each clue thoroughly is one of the vital skills in solving problems using logical reasoning. It is very important that students take the time to read the clues and understand each one. They can then decide where to begin in solving the problem.

Decide where to begin

Often the clues need to be dealt with in a different order to the order in which they are presented.

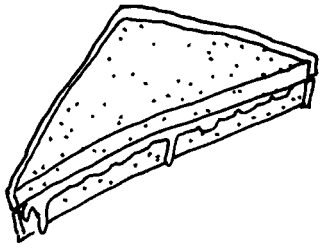
Example

Julie, Yushiko and Sam and are each about to eat a sandwich for lunch. On the plate there is a tomato sandwich, a honey sandwich and a peanut butter sandwich. Use the following clues to work out which sandwich belongs to each person.

· Julie's sandwich has salt and pepper on it.
· Sam is allergic to nuts.
· Yushiko hates sweet things.

It is probably easiest to begin with the fact that Sam is allergic to nuts, which means the peanut butter sandwich cannot be his. Julie's sandwich has salt and pepper on it, so it is not likely to be the peanut butter sandwich. She must have the tomato sandwich. This leaves Sam with the honey sandwich (which Yushiko would not have had, since she hates sweet things).

So Julie has the tomato sandwich, Sam has the honey sandwich, and Yushiko has the peanut butter sandwich.



Draw up a grid listing the names and choices

Drawing up a grid can be a convenient way of visualising the information presented in a problem, and clarifying the steps taken to solve it.

For the previous example, the grid would look like this:

	tomato	honey	peanut butter
Julie			
Yushiko			
Sam			

Once drawn up, a grid can be systematically marked up with ticks and crosses (or true and false, or yes and no, or whatever is most suitable) to clarify each step and visualise the information that has been given.

	tomato	honey	peanut butter
Julie	✔	✘	✘
Yushiko	✘	✘	✔
Sam	✘	✔	✘

Teaching Examples Using Logical Reasoning

Example 1

Four children, Alan, Bernard, Tammi and Sue, own four dogs called Tom, Bubby, Susie and Arty. Use the information below to work out which dog belongs to which owner.

- Nobody owns a dog whose name starts with the same letter as their own.
- Tammi and Bubby's owner are friends.
- Bernard's sister owns Arty.
- Alan gives Susie's owner a bone.
- Susie's owner said Tammi's dog was aggressive.
- Alan wished he had a dog like Tom.

Understanding the problem

What do we know?

There are four children and they each owned a dog.
Each dog's name starts with a different letter to their owner's name.

What do we need to find out?

Questioning: Which dog belongs to which person? Which clue will be a good starting point?

Communicating a solution

Draw up a 5 x 5 grid. Use the symbols ✔ and ✘ to represent the different pieces of information contained in the clues. For example, since we know that none of owners' names begin with the same letter as their dogs' name, we can immediately put crosses in the boxes that link Alan and Arty, Bernard and Bubby, Tammi and Tom, and Sue and Susie.

Filling in the other pieces of information gives us this grid:

	Tom	Bubby	Susie	Arty
Alan	✘	✔	✘	✘
Bernard	✘	✘	✔	✘
Tammi	✘	✘	✘	✔
Sue	✔	✘	✘	✘

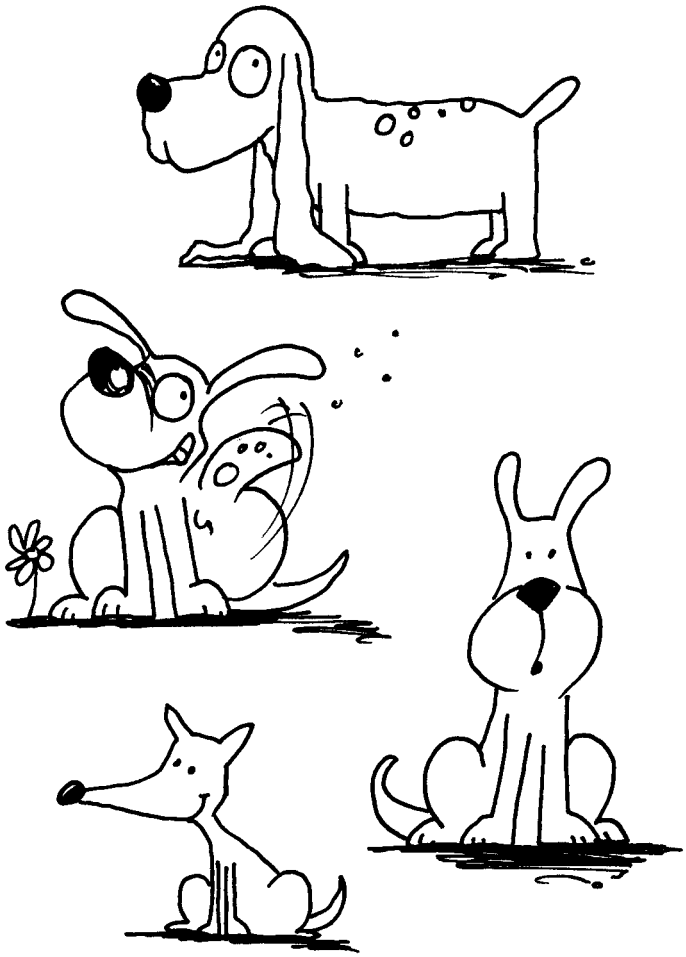
From this we can see that Alan owns Bubby, Bernard owns Susie, Tammi owns Arty and Sue owns Tom.

Reflecting and generalising

Think about the strategy of creating a grid and using ticks and crosses to represent the information provided. Is this a logical approach to use when a number of statements are made? Is there another way to solve this problem?

Extension

Encourage students to write simple problems of their own that include a number of logical statements and can be approached using this technique.



Example 2

Twenty-four boarders live at a school where the rest of the students are day students. The boarding school is built in the shape of a square, divided into nine rooms. The students sleep three to a room, and the teacher sleeps in a room in the middle in order to supervise the students. At night the teacher has to ensure that there are nine students on each side of the square.

One night some of the boarders slipped out. How did the students manage to deceive their teacher? What is the greatest number of students that could go out at a time and still leave nine students on each side of the square?

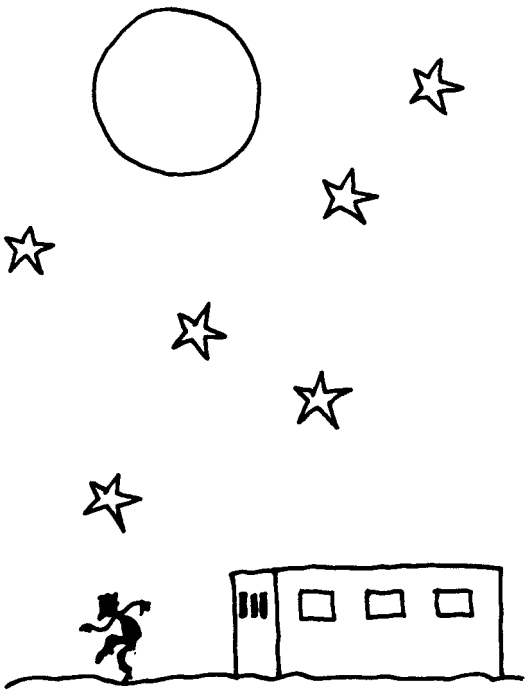
Understanding the problem

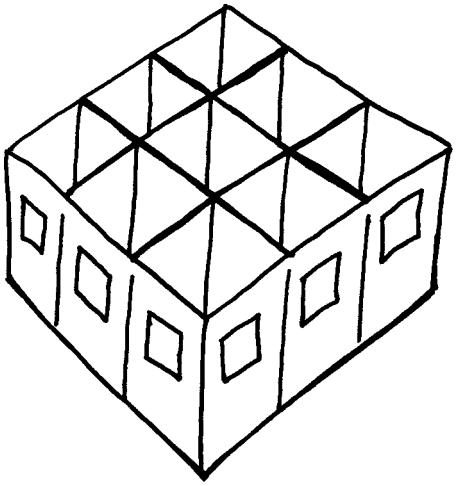
What do we know?

There are 24 boarders. Students sleep three to a room. The teacher sleeps in the middle room. The boarding school is built in the shape of a square. There have to be 9 students on each side.

What do we need to find out?

Questioning: How did they manage to deceive the teacher?
What is the greatest number of students that could go out at a time and still do this?





Communicating a solution

The corner rooms are counted in both directions, so more students should be placed in those rooms.

5	1	3
1		1
3	1	5

In this way, 4 students could sneak out and still leave nine students on each side.

Six is the greatest number that can go out as 18 students must be left inside.

9	0	0
0		0
0	0	9

Reflecting and generalising

The first logical step untilises the knowledge that the corner students are counted twice. To allow more students to sneak out, more of the students that remain should therefore be placed in the corner rooms. This concept should be applied to other problems where there are limited numbers that can be used for maximum advantage.

Extension

Ask students to calculate the problem changing the number of students that are boarders and the number that need to stay at home. Are they able to apply the same concepts to different numbers?

Example 3

Five children, Andy, Jocelyn, Nick, Rob and Kahlee, brought their favourite fruits to school. They brought peaches, apples, grapes, apricots and passionfruit. Use the clues below to work out which person brought which fruit.

- Jocelyn's favourite fruit is passionfruit. Andy's favourite fruit starts with the same letter.
- Nick's and Rob's fruit don't start with the same letter.
- Nick likes apples.

Understanding the problem

What do we know?

Five children brought their favourite fruit to school.
They brought peaches, apples, grapes, apricots and passionfruit.

What do we need to find out?

Questioning: What fruit did each person bring?

Communicating a solution

Draw up a grid that is 6 x 6. Write the names of the children down the left hand side of the grid and the names of the fruit along the top of the grid.

	peaches	apples	passionfruit	grapes	apricots
Andy	✔	✘	✘	✘	✘
Jocelyn	✘	✘	✔	✘	✘
Nick	✘	✔	✘	✘	✘
Rob	✘	✘	✘	✔	✘
Kahlee	✘	✘	✘	✘	✔

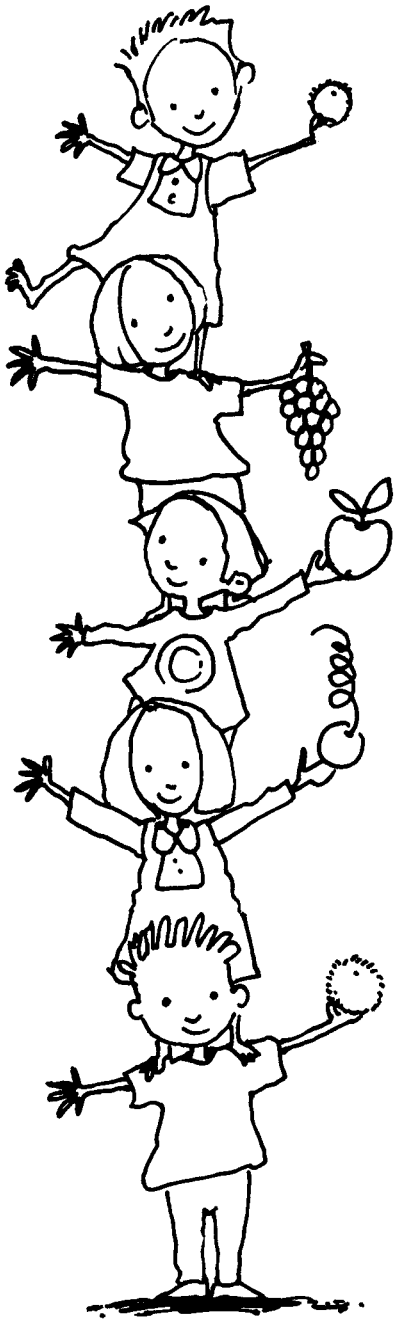
Andy brought the peaches, Jocelyn brought the passionfruit, Nick had the apples, Rob had the grapes, and Kahlee had the apricots.

Reflecting and generalising

The information was checked off using a system of ticks and crosses that helped with the visualisation and allowed us to put the clues into a concrete form.

Extension

Ask students to prepare a logical problem using six people and six objects. They will need to ensure that they have included enough clues to allow the problem to be solved.



BLM Using Logical Reasoning

★ Understanding the problem

List the facts that will be important in finding the solution:

★ What do you need to find out?

What questions do you have? What are you uncertain about? Is there any unfamiliar or unclear language?

★ Planning and communicating a solution

Will you work logically? In what order will you use the clues? Will you find one that can be used as a starting point and from which the others can be logically thought out? Will you use a grid to help visualise the clues? Will you use objects to stand for the people and places in order to help visualise the situation?

★ Reflecting and generalising

Did the strategy work as planned? Will you be able to apply this method of problem solving to other similar problems? Would a different method have worked better for you for this problem?

★ Extension

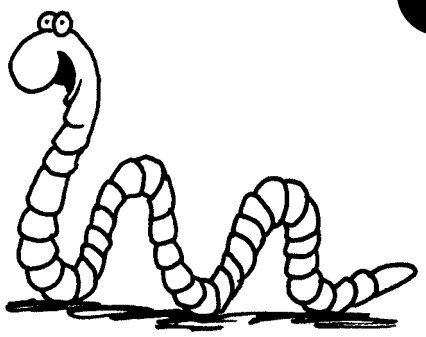
How can this strategy be applied to more complicated problems involving additional factors?

Problem 1

Measurement

Level 1

In the story Harry was reading, a forever-hungry worm ate everything in sight. It ate so much that, incredibly, it doubled its size every day. If it took 11 days for the worm to be fully grown, how long did it take the worm to get to half its full-grown size?



Problem 2

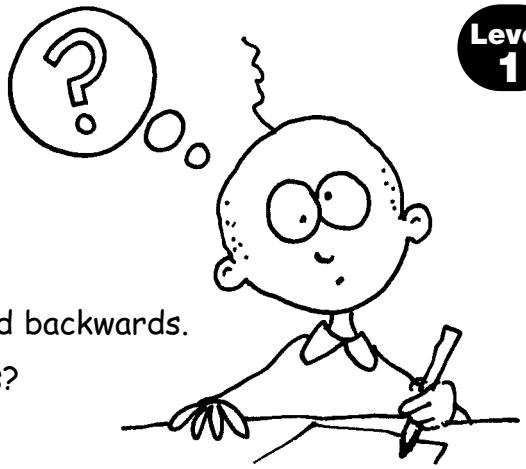
Number 1 2 3

Level 1

There are three different numbers.

- They are all four-digit odd numbers.
- The digits of each number add up to 14.
- None of the numbers can be divided by 5.
- These numbers read the same forwards and backwards.

Can you guess the numbers using these clues?



Problem 3

Number 1 2 3

Level 1

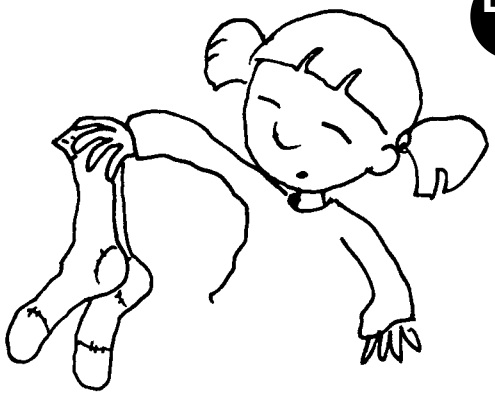
When Clarice opened her book, she saw two numbered pages. The sum of these two pages was 317. What would the next page number be?

Problem 4

Number 1 2 3

Level 1

I have 30 socks in my drawer and they vary in colour. I have 12 black, 10 blue, 2 brown and 6 grey socks. If I close my eyes and pull socks out at random, how many socks must I pull out before I can be certain to have a matching pair?

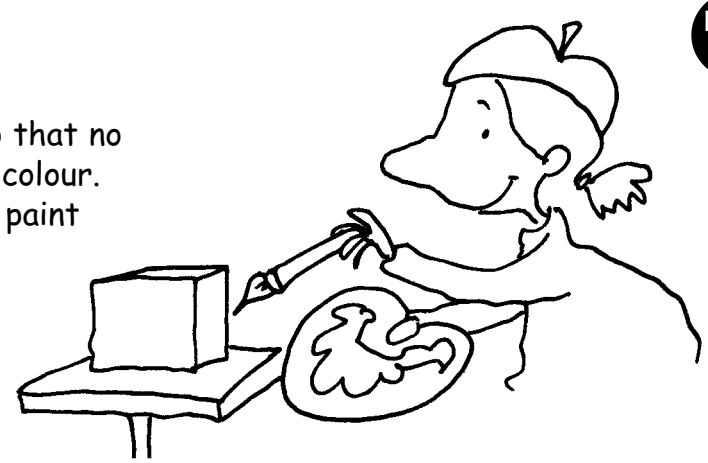


Problem 5

Space

Level 1

Rani wants to paint a cube so that no opposite faces are the same colour. What is the least number of paint colours she should buy?

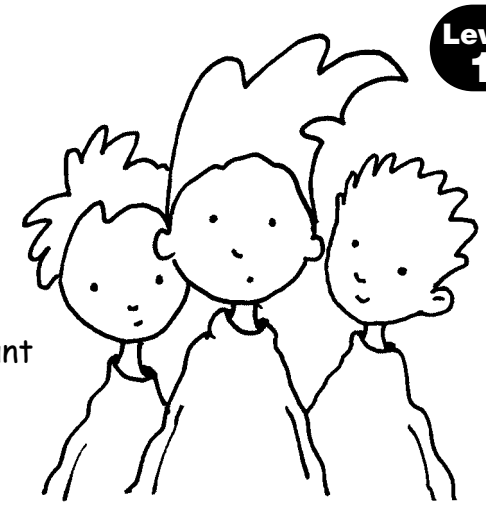


Problem 6

Number 1 2 3

Level 1

When Tristram, Evelyn and Carol leave school, they want to work in one of the following professions: as a teacher, an engineer or a computer programmer. None of them wants to work in a profession starting with the same letter as their name. Evelyn's mother is the aunt of the person who wants to become a teacher. What does each person want to become?

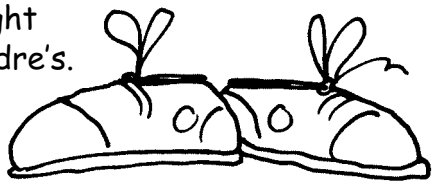


Problem 7

Measurement

Level 2

Five students left their sneakers in the change rooms. Each wore the same brand of sneakers, but in a different size. The sneakers were sizes 2, 3, 4, 5 and 6.
Tensu knew that his were the smallest. Karen thought hers were bigger than Chan's but smaller than Deirdre's. Brad knew his were the largest.
Which size sneaker did each student have?



Problem 8

Number 1 2 3

Level 2

There are 105 animals living on Farrelly's Farm. There are half as many rabbits as guinea pigs. There are half as many geese as sheep. There are three times as many geese as guinea pigs. How many of each farm animal are there?

Problem 9

Measurement

Level 2

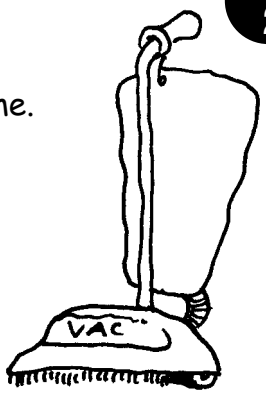
The students at camp hold a running race. Use these clues to work out their positions at the end of the race.

Peter was extremely fast and was the first boy to complete the race. Lisa came in just before Jonathan. Jonathan finished after Catherine. Michelle ran very well and managed to beat Catherine. Peter finished after Michelle but before Catherine.

Problem 10

Space

Level 2



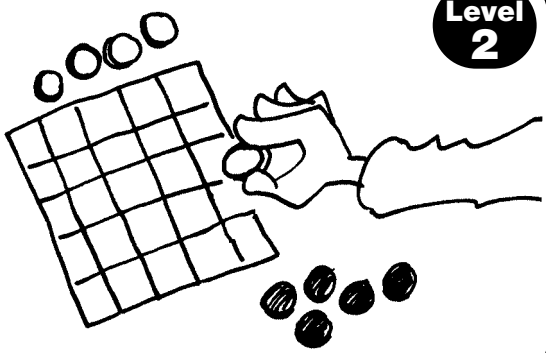
Thomas and Trina help their parents with the chores at home. Complete a grid to show the order in which each completes their chores.

- Thomas vacuums the carpet after Trina dries the dishes.
- The person who cleans the bathroom does this first.
- The dishwasher is emptied before the mail is collected. Different people do these tasks.
- The last thing that Trina does is take the dog for a walk.
- Trina does not walk the dog at the same time as the carpet is being vacuumed.

Problem 11

Number 1 2 3

Level 2



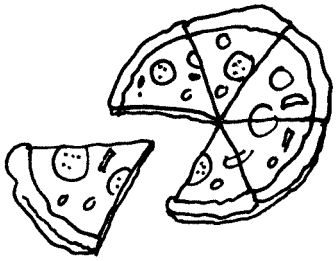
Draw up a five by five square chart. You will need five sets of coloured counters, with five of each colour. Place the coloured counters on the chart so that there is one of each colour in each row, column and diagonal.

Problem 12

Number 1 2 3

Level 2

Carter, Jackie, Despina and Fred ordered pizza slices. They could choose from peperoni, vegetarian, anchovies and cheese. Match each person with the pizza they ordered.



- Carter hates all seafood.
- Jackie can't eat milk products.
- Despina loves meat.
- Fred is a vegetarian.

Problem 13

Number 1 2 3

Level 3

Carmen bought five tickets to watch a game of footie with her friends. The tickets were numbered in order. When the numbers are added together, they total 110. What were the ticket numbers.

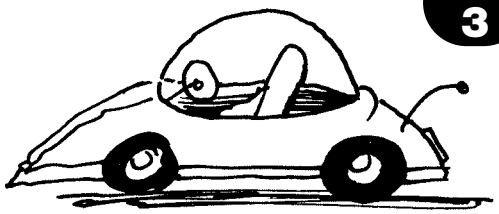
Problem 14

Number 1 2 3

Level 3

Five cars are in a race. Their numberplates are:

1733 5824 9762 6465 7525



- The car with the highest number finished last.
- When the digits in the numberplates of the cars that came first and second are added, they give the same total.
- The cars that came in second and third both have an odd number as the last digit.
- The cars that came in third and fifth both bear numbers that are multiples of five.

Use these clues to work out how the racing cars placed in the race.

Problem 15

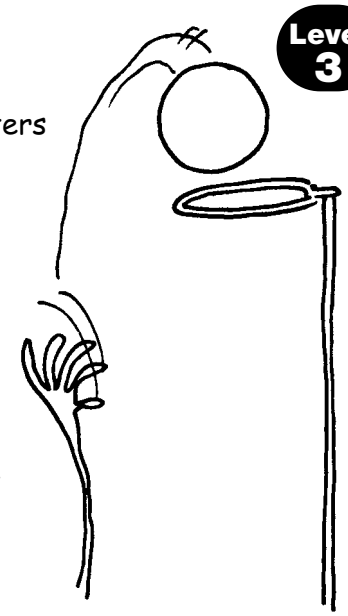
Number 1 2 3

Level 3

Jennifer, Jackie, Tandi, Lindsay, Sara and Lesley are all shooters in different netball teams. Between them one Saturday they scored a total of 33 goals. Each girl scored a different number of gals and each girl scored at least one goal.

- Jennifer was the top scorer and scored three more than Jackie.
- Tandi scored one less than Lindsay but their total was the same as Jennifer's and Jackie's scores added together.
- Lesley beat Sara's scoring level but their scores together equalled the number of goals Lindsay had scored on her own.

Use these clues to work out how many goals each girl scored.



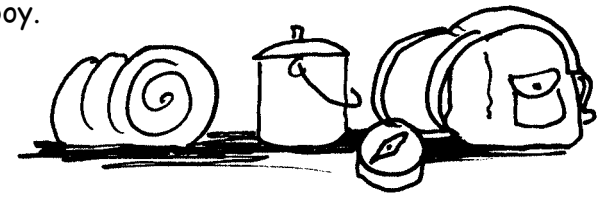
Problem 16

Number 1 2 3

Level 3

Jane, Graham, Rebecca and Paul are planning their camping holiday this year. Each wants to visit a different place. One wants to go to Plet, another to Goshen, the third to Minto and the last to Grahamstown. Use the clues to work out who wants to go where.

· No-one wants to visit the place starting with the same letter as their name.
· The person who wants to visit Minto is a boy.
· A boy wants to visit Goshen.
· Graham likes Minto and Goshen.
· Jane did not want to visit Plet this year.



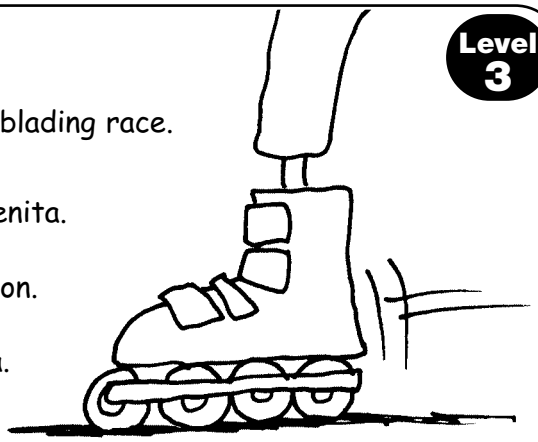
Problem 17

Space

Level 3

Dirk, Arthur, Benita, Caren and Edie had a roller-blading race. Use the clues to work out their finishing order.

· Caren did not finish just before or just after Benita.
· Dirk finished just in front of Benita.
· Neither Arthur nor Benita finished in last position.
· Caren wasn't in first or last place.
· Dirk completed the route just in front of Benita.
· Edie didn't finish just before or after Caren.



Problem 18

Space

Level 3

Four friends—Rudi, Maya, Jeff and Frances—went out for lunch. When they sat down at the table, each was wearing a different coloured shirt. Their shirts were black, yellow, white and blue. Each ordered something different for lunch: a hamburger, a pizza slice, a hot dog and a sandwich. Work out who sat in which seat, which colour shirt they were wearing and what each ordered to eat.

The friend in black sat opposite the friend in white who ordered the slice of pizza. The friend in yellow was not sitting in a westerly or southerly seat. Jeff sat in the northerly seat. The person who sat opposite Jeff had a hamburger to eat. The person in the easterly seat had a hot dog and wore a black shirt. This wasn't Frances, who was wearing a blue shirt. Maya sat in the westerly seat.

Answers to Task Cards

Problem 1

10 days. As it doubles its size every day and is fully grown by day 11, it would be half its full-grown size on day 10.

Problem 2

The numbers are 1661, 3443 and 7007.

Problem 3

The next page number is 160.
Clarice knows that one of the pages must be one higher than the other page number. Dividing 317 by two gives 158 with one remaining. So the page numbers she can see must be 158 and 159.

Problem 4

I would need to pull out 5 socks. Because there are four different colours, even if I pull out a different coloured sock each time on the first four goes, the fifth has to provide a matching sock.

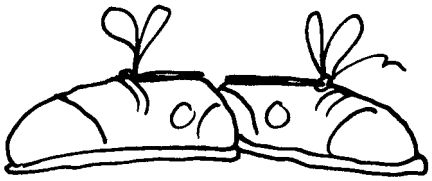
Problem 5

Two colours. Supply students with the net of a cube so that they can experiment with colours. Point out that the problem states opposite faces cannot be the same colour, but that this does not apply to adjacent faces.

Problem 6

	Teacher	Engineer	Computer programmer
Tristram	✔	✘	✘
Evelyn	✘	✔	✘
Carol	✘	✘	✔

(Because you are told that Evelyn's mother is the aunt of the person who wants to be the teacher, you know that Evelyn herself does not want to be a teacher.)



Problem 7

	2	3	4	5	6
Tensu	✔	✘	✘	✘	✘
Karen	✘	✘	✔	✘	✘
Chan	✘	✔	✘	✘	✘
Deirdre	✘	✘	✘	✔	✘
Brad	✘	✘	✘	✘	✔

So Tensu wears size 2, Chan size 3, Karen size 4, Deirdre size 5 and Brad size 6.

Problem 8

5 rabbits, 10 guinea pigs, 30 geese, 60 sheep. Have students start by putting the animals in order from least to most.

Problem 9

	1st	2nd	3rd	4th	5th
Peter	✘	✔	✘	✘	✘
Lisa	✘	✘	✘	✔	✘
Jonathan	✘	✘	✘	✘	✔
Catherine	✘	✘	✔	✘	✘
Michelle	✔	✘	✘	✘	✘

Problem 10

Thomas	Trina
cleans the bathroom	dries the dishes
vacuums the carpet	empties the dishwasher
collects the mail	walks the dog

Problem 11

There are several solutions. One of them is:

R	B	G	W	Y
W	Y	R	B	G
B	G	W	Y	R
Y	R	B	G	W
G	W	Y	R	B

R = red
B = blue
G = green
Y = yellow
W = white

Problem 12

	peperoni	vegetarian	anchovies	cheese
Carter	✘	✘	✘	✔
Jackie	✘	✘	✔	✘
Despina	✔	✘	✘	✘
Fred	✘	✔	✘	✘

Problem 13

The ticket numbers are 20, 21, 22, 23, 34. The average number for each ticket is 22 (110 ÷ 5). Work forwards and backwards from 22.

Problem 14

First	5824
Second	7525
Third	6465
Fourth	1733
Fifth	9762

Problem 15

Jennifer	8 goals
Lindsay	7 goals
Tandi	6 goals
Jackie	5 goals
Lisa	4 goals
Sara	3 goals

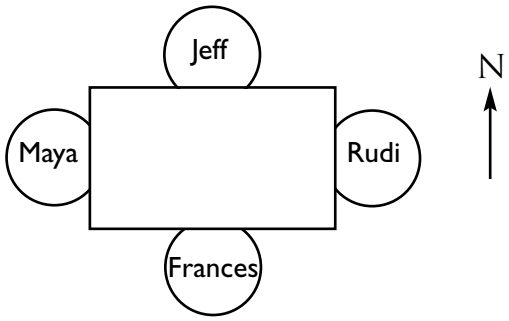
Problem 16

	Plet	Goshen	Minto	Grahamstown
Jane	✘	✘	✘	✔
Graham	✘	✘	✔	✘
Rebecca	✔	✘	✘	✘
Paul	✘	✔	✘	✘

Problem 17

	1st	2nd	3rd	4th	5th
Dick	✘	✘	✔	✘	✘
Arthur	✔	✘	✘	✘	✘
Benita	✘	✘	✘	✔	✘
Caren	✘	✔	✘	✘	✘
Edie	✘	✘	✘	✘	✔

Problem 18



Rudi, black shirt, hot dog
Maya, white shirt, pizza
Jeff, yellow shirt, sandwich
Frances, blue shirt, hamburger