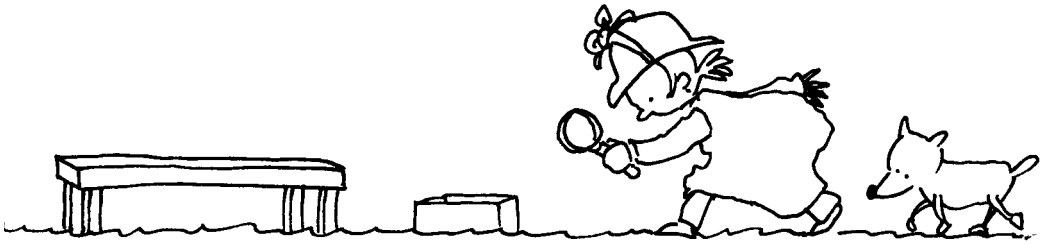
Blake's Topic Bank

UNIT P11 ■
Problem Solving
Upper Primary

Problem Solving
Analysing and Investigating

by **Sharon Shapiro**



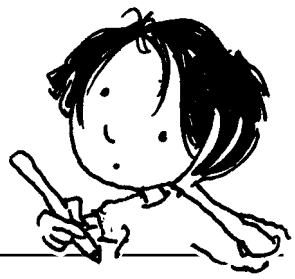
This unit contains:

- Teaching notes
- 3 teaching examples
- 1 BLM
- 18 task cards
- Answers

Problem Solving
Analysing and Investigating

Sharon Shapiro

Upper Primary



The Problem Solving Process

It is important that students follow a logical and systematic approach to their problem solving. Following these four steps will enable students to tackle problems in a structured and meaningful way.

Step 1: Understanding the problem

- ❖ Encourage students to read the problem carefully a number of times until they fully understand what is wanted. They may need to discuss the problem with someone else or rewrite it in their own words.
- ❖ Students should ask internal questions such as, what is the problem asking me to do, what information is relevant and necessary for solving the problem.
- ❖ They should underline any unfamiliar words and find out their meanings.
- ❖ They should select the information they know and decide what is unknown or needs to be discovered. They should see if there is any unnecessary information.
- ❖ A sketch of the problem often helps their understanding.

Step 2: Students should decide on a strategy or plan

Students should decide how they will solve the problem by thinking about the different strategies that could be used. They could try to make predictions, or guesses, about the problem. Often these guesses result in generalisations which help to solve problems. Students should be discouraged from making wild guesses but they should be encouraged to take risks. They should always think in terms of how this problem relates to other problems that they have solved. They should keep a record of the strategies they have tried so that they don't repeat them.

Some possible strategies include:

- ❖ Drawing a sketch, graph or table.
- ❖ Acting out situations, or using concrete materials.
- ❖ Organising a list.
- ❖ Identifying a pattern and extending it.
- ❖ Guessing and checking.
- ❖ Working backwards.
- ❖ Using simpler numbers to solve the problem, then applying the same methodology to the real problem.
- ❖ Writing a number sentence.
- ❖ Using logic and clues.
- ❖ Breaking the problem into smaller parts.

Step 3: Solving the problem

- ❖ Students should write down their ideas as they work so they don't forget how they approached the problem.
- ❖ Their approach should be systematic.
- ❖ If stuck, students should reread the problem and rethink their strategies.
- ❖ Students should be given the opportunity to orally demonstrate or explain how they reached an answer.

Step 4: Reflect

- ❖ Students should consider if their answer makes sense and if it has answered what was asked.
- ❖ Students should draw and write down their thinking processes, estimations and approach, as this gives them time to reflect on their practices. When they have an answer they should explain the process to someone else.
- ❖ Students should ask themselves 'what if' to link this problem to another. This will take their exploration to a deeper level and encourage their use of logical thought processes.
- ❖ Students should consider if it is possible to do the problem in a simpler way.

Investigative group work is vitally important. Students will learn from each other as they explore ideas verbally.

The first step for students carrying out investigative work is to go through a process of analysing what they know and what they need to know in order to solve the problem. Often students immediately assume that reference material is needed, but with practice students will realise that simple steps and their own experience can result in a reasonable estimate.

Problems that involve group investigations provide opportunities for students to pool their existing knowledge. Instead of having to refer to outside sources, students will find that much of the necessary knowledge already exists within the group.

There are several skills and strategies that are useful for investigative problems.

Estimation

Estimation focuses on a mathematical concept without asking students to become too involved in the calculation of the algorithm. Once students have discussed and trialed an estimate, it may be altered.

Stress the importance of estimates as useful checks on the progress of solving a problem. For example, when using a calculator students frequently may press a wrong key, or not press properly, or accidentally press a key twice. If students have begun with a rough estimate, they will be aware that their answer is far off the original estimate, and that this may be due to simple calculating error. The calculation process can then be repeated.

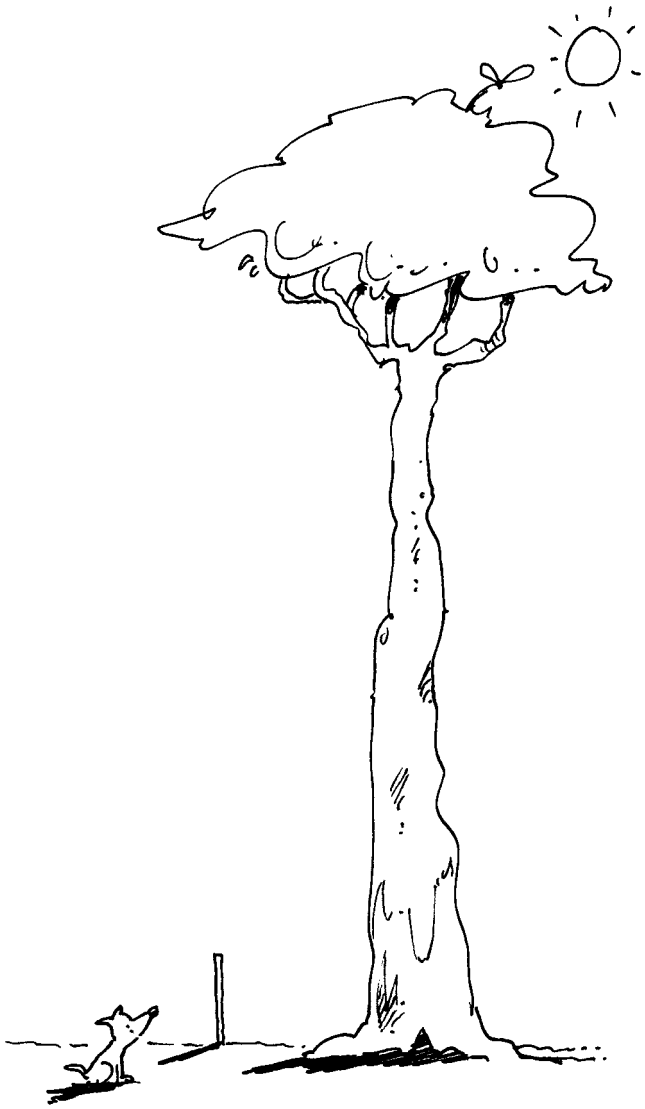
As a practice exercise, give students five numbers and ask them to estimate the average without adding the numbers. Follow this with exercises where students estimate the height of a door, width of a desk, width of a window, perimeter of a desk or area of classroom before actually calculating. With practice, students' initial estimates should become more accurate.

Quick mental computation

Practice in rounding off, doubling, patterning, tables, multiples and factors will assist students with the skills needed to calculate more accurately.

Planning an approach to gather information

Students will need to consider whether the task involves measurement, observation, a survey, a drawn diagram. They should explore the different methods, processes and strategies that can be used to gather information, and decide how they want to write up or display what has been discovered.



Example 1

Each day, vehicles continually stream past the school. How long will it take for a million cars to pass?

Understanding the problem

What do we know?

Cars rush past the school each day. Eventually a million cars will pass.

What do we need to find out?

Questioning: What can we do with what we know? How can we prove what we know or find out? In what order will we carry out the investigation? What processes or strategies will we use?

Communicating a solution

One possible solution:

Students believe there are two peak traffic times, from 8:00–9:00 am and 6:00–7:00 pm. During the remainder of the day, cars do not pass as frequently.

Plan: Survey the traffic during the peak hours, and at other times during the day. Tally the number of vehicles that pass during a ten-minute period, and multiply by six to obtain the number of cars per hour.

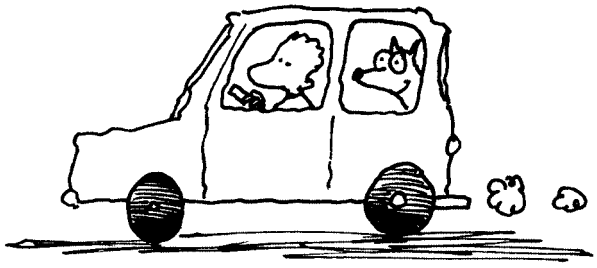
Peak hour

8:00–9:00 am	250 cars in a ten-minute period, ie $250 \times 6 = 1500$ cars/hour
6:00–7:00 pm	Assume it is the same, 1500 cars/hour.

Other times

1:00–2:10 pm	60 cars in a ten-minute period, ie $60 \times 6 = 360$ cars/hour

The period between midnight and 6:00 am would be very quiet—estimate 30 cars per hour during this time.



The flow of traffic in one 24-hour period can be recorded as follows:

Time	Number of cars
7:00–8:00 am	360
8:00–9:00 am	1800
9:00–10:00 am	360
10:00–11:00 am	360
11:00–12:00 noon	360
12:00–1:00 pm	360
1:00–2:00 pm	360
2:00–3:00 pm	360
3:00–4:00 pm	360
4:00–5:00 pm	360
5:00–6:00 pm	360
6:00–7:00 pm	1800
7:00–8:00 pm	360
8:00–9:00 pm	360
9:00–10:00 pm	360
10:00–11:00 pm	360
11:00–12:00 am	360
12:00–1:00 am	30
1:00–2:00 am	30
2:00–3:00 am	30
3:00–4:00 am	30
4:00–5:00 am	30
5:00–6:00 am	30
6:00–7:00 am	360

Total number of cars in one day = 9540

1 000 000 cars divided by 9540 cars/day= 104·8 days until a million cars pass.

Reflecting and generalising

In the process of the investigation, students have used the strategies of observation (tallying the peak and average traffic flow times), generalisation (applying these tallies to other peak and non-peak times), and assumption (making an estimate for the low traffic flow time). These strategies will be useful in other similar problems.

Extension

It is important that students develop their own investigations about things they see happening around them. Doing this will involve them in exploring different thinking patterns.

Example 2

Imagine you want to make an exact copy of one of the tables in the classroom. Find out and record all the attributes that make this table unique so that the maker can copy it for you.

Understanding the problem

What do we know?

We want to have a table made. We want it to be the same as a particular table in the classroom. We need to give the maker clear instructions about what we want.

What do we need to find out?

Questioning: What are the dimensions needed? How will the dimensions be measured? What level of accuracy is needed? What other information will we need to give? In what order will we carry out the investigation? What processes or strategies will we use? How can we best record the information needed?

Communicating a solution

Have all dimensions such as height, length, and width been measured? Have all the parts, for example the legs of the table and the thickness of the top, been measured? Have rulers, tape measures, handspans or other processes of measuring been used?

Following is an example of how the information could be presented:

Dimensions of the table
Height: .....
Length: .....
Width: .....
Thickness of the table top: .....
Corners (pointy or rounded): .....

Number of legs: .....
Shape: .....
Length: .....
Width: .....

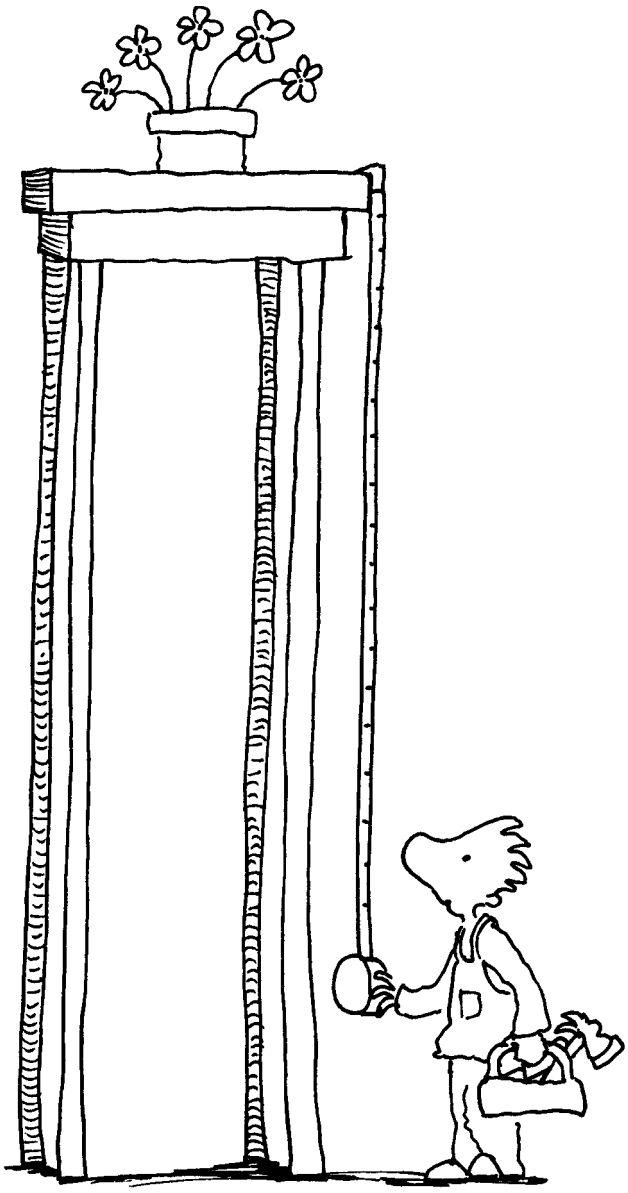
Type of wood: .....
Colour: .....
Any other relevant information: .....
.....
.....

Reflecting and generalising

Was a systematic approach used in gathering the information? Was information recorded as it was obtained, in a clear format? This type of problem can be used to introduce students to systematic ways of recording information they have gathered. It can also be used in investigative group work, where the delegation of particular aspects of information gathering can speed up the process.

Extension

This type of problem can be extended to feature items with more complicated attributes.



Example 3

There is a very tall tree in the playground whose height your teacher wants you to measure. You do not have anything tall enough to use to measure the tree.

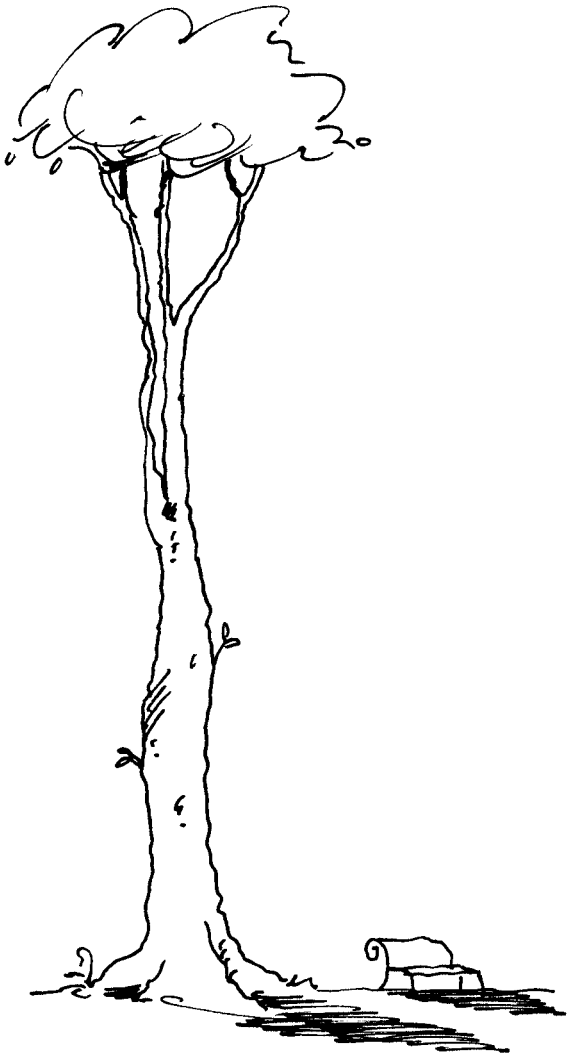
Understanding the problem

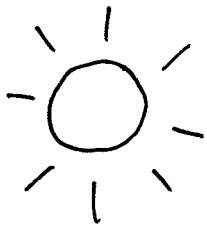
What do we know?

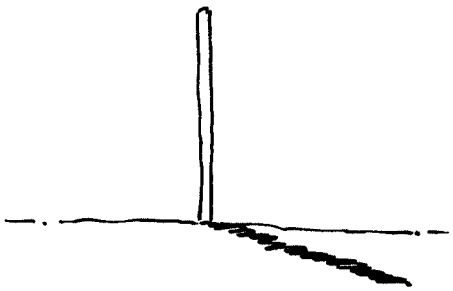
The tree is much taller than we are. We do not have anything large or high enough to use to measure the height.

What do we need to find out?

Questioning: What can we do with what we know?
How can we prove what we know or find out?
In what order will we carry out the investigation?
What processes or strategies will we be using?







Communicating a solution

There are many ways to solve this problem. Here are two possibilities:

- Send a friend to climb up the tree with a length of rope. When you think he or she is half-way up, have them let down the rope so that the bottom touches the ground. Have them mark the rope at the point where they are holding it, which marks half the tree's height. This can be measured and doubled when the climber returns to the ground.
- Push a stick into the ground early in the day. When the length of the stick's shadow is equal to the height of the stick, the length of the shadow of the tree will be equal to the height of the tree. Measure the tree's shadow, and you have the tree's height!

Reflecting and generalising

Did the strategy work as planned? Is there a better or more accurate method that could have been used?

Extension

Have students develop their own investigative problems that involve things they see happening around them. Students could work in groups, each of which could set a problem for another group to solve.

BLM Analysing and Investigating

★ Understanding the problem

What do you know? List the facts that will be important in finding the solution:

★ What do you need to find out?

What processes or strategies will you use? What equipment do you need for the investigation? How accurate do all the calculations need to be? Would an estimate be a satisfactory answer? Do you understand all aspects of the problem? Is there any unfamiliar or unclear language?

★ Planning and communicating a solution

Read the problem and decide where to start. Predict what you believe the outcome will be. How many different ways can you conduct the investigation? Which method will you use? How will you gather the information? How will you record the information? Try to work methodically, thinking all the way through one aspect or part.

★ Reflecting and generalising

Did the strategy work as planned? Will you be able to apply this method of problem solving to other similar problems? Would a different method have worked better for you for this problem?

★ Extension

How can this strategy be applied to more complicated problems involving additional factors?

PROBLEM SOLVING TASK CARDS - Analysing and Investigating

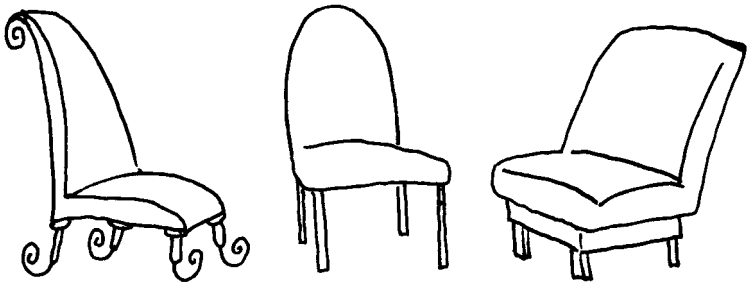
Problem 1

Measurement

Level 1

Investigate the most common height for chairs.

Try to suggest a reason for this. Predict before you begin. Think about how you will collect and display your information.



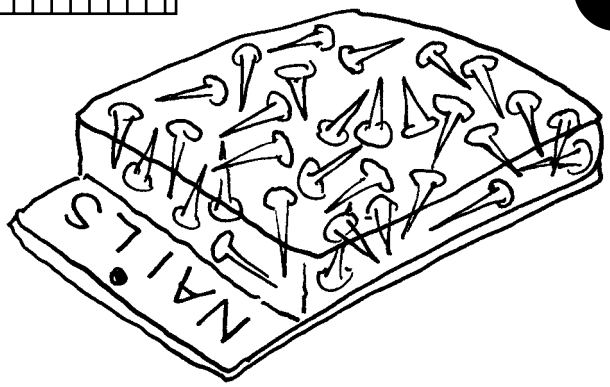
Problem 2

Number 1 2 3

Measurement

Level 1

How many nails in a packet? Examine the nails purchased in a packet to estimate and then calculate the quantities.

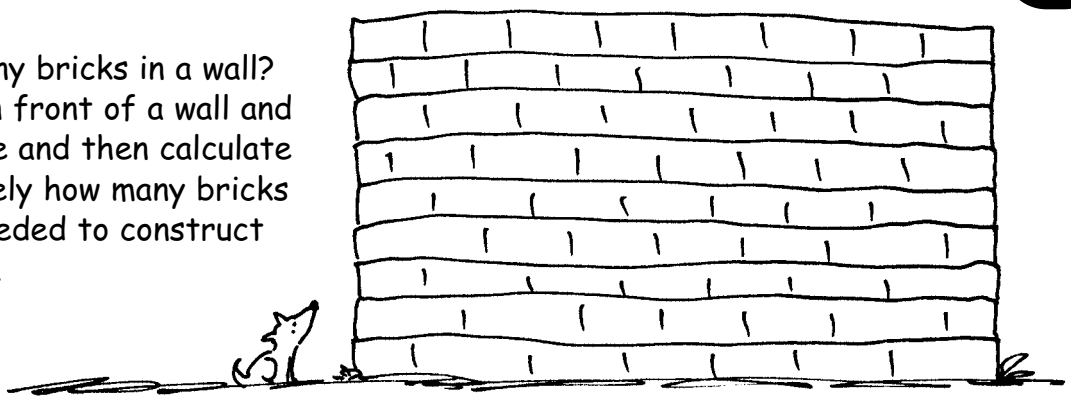


Problem 3

Number 1 2 3

Level 1

How many bricks in a wall? Stand in front of a wall and estimate and then calculate accurately how many bricks were needed to construct the wall.

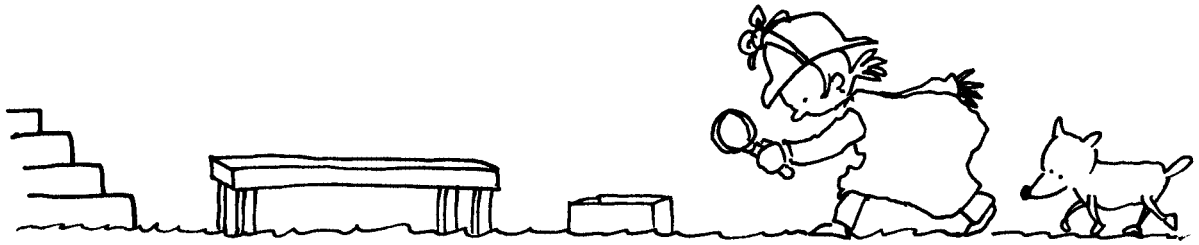


Problem 4

Space

Level 1

Investigate how many 3-dimensional shapes can be found around the school.

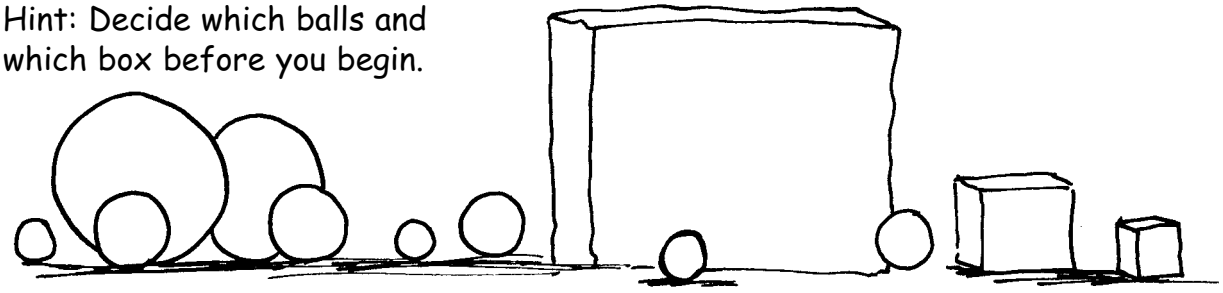


Problem 5

Space

Level 1

How many balls will fill a box?
Hint: Decide which balls and which box before you begin.



Problem 6

Measurement

Level 2

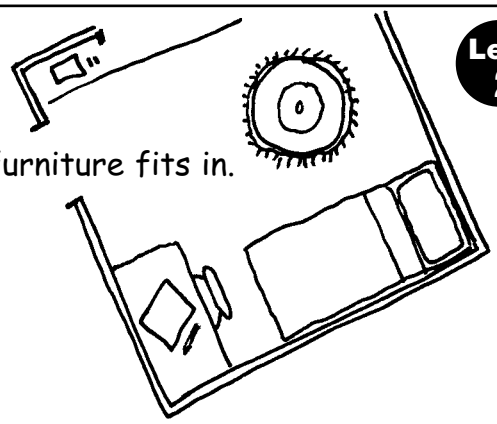
After a rugby match at the Olympic centre in Homebush, 49 000 Sydney fans left in the bins 864 000 empty drink cans. These cans fill an amazing 24 garbage trucks.

What is the mass of an empty cold drink can? What was the mass of the cans carried by the trucks?

Problem 7

Measurement

Level 2



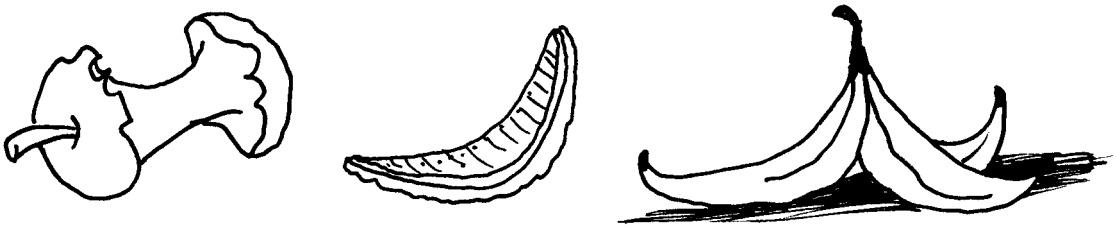
Redesign your bedroom. Make sure all your furniture fits in.
Draw the new arrangement to scale.
Remember:
- Leave space to open doors and drawers.
- Leave space to play games, if possible.
- Make sure you can easily open the window.

Problem 8

Number 1 2 3

Level 2

Work out how many pieces of fruit would be eaten by the pupils in your school over a period of one week.

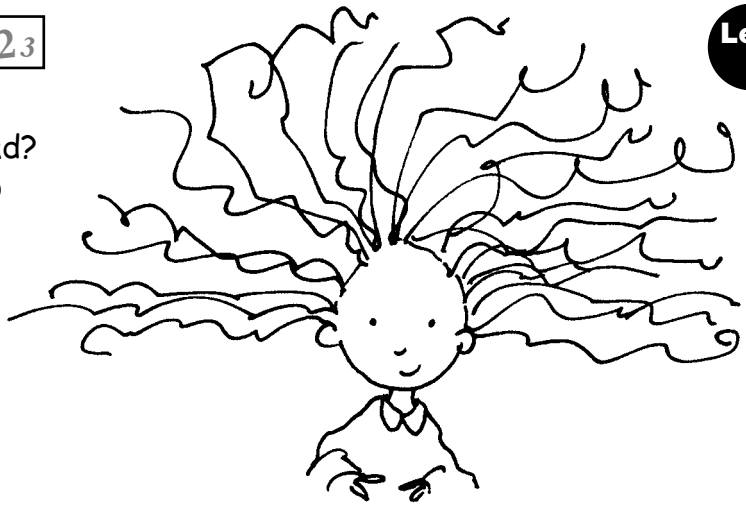


Problem 9

Number 1 2 3

Level 2

How many hairs on your head? Examine an average head to estimate and then calculate the number of hairs on a person's head.

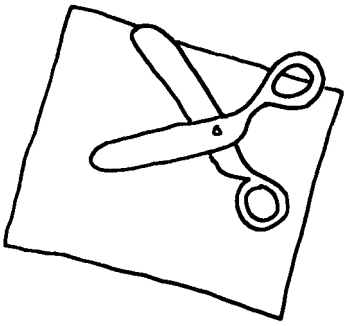


Problem 10

Space

Level 2

How many different ways are there to cut a square into four equal parts? Predict before you begin. Have several squares of paper available.

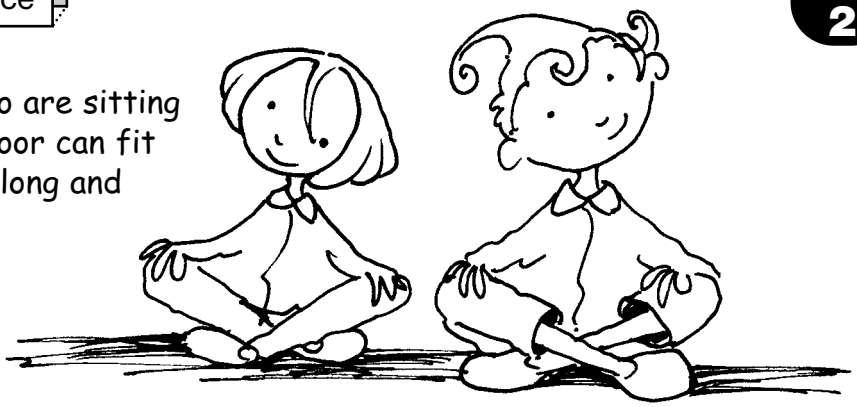


Problem 11

Space

Level 2

How many children who are sitting cross-legged on the floor can fit into a space 4 metres long and 2 metres wide?



Problem 12

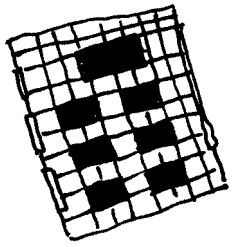
Measurement

Level 3

Work with a group of students to redesign the classroom. Everything that you presently see in your classroom should remain. Draw to scale, after discussing what would be a suitable scale.

Make sure you:

- leave space between objects of furniture
- include sufficient tables and chairs to seat all students
- ensure there is space for group work.

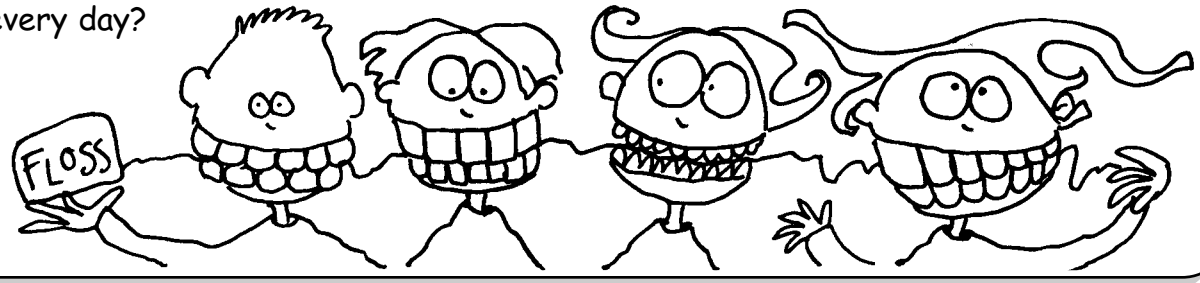


Problem 13

Measurement

Level 3

How long would a dental floss container last if a family of four flosses once every day?

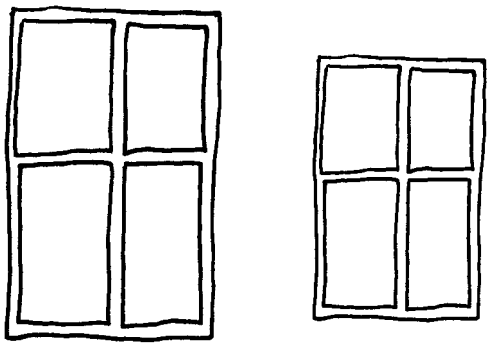


Problem 14

Measurement

Level 3

How many square metres of glass would be needed to replace all the windows at your school? How many windows are there and how large is each pane of glass? Is there a standard size?



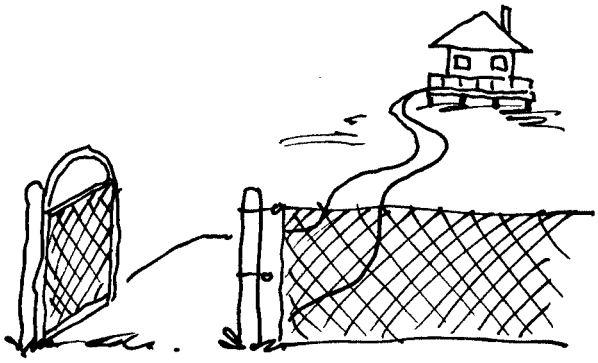
Problem 15

Number 1 2 3

Measurement

Level 3

Estimate and work in groups to calculate how long a money trail stretching from the school gate to the classroom would be, and how much money would be raised with this trail. You will need a calculator to help with the calculations. How would using different coins affect the answer?



Problem 16

Number 1 2 3

Level 3

Over a period of a week, find out the total cost of food bought from the canteen by your class. You may need a price list to help students who do not recall costs of items they have purchased.

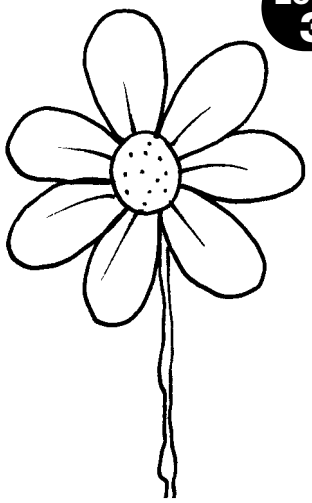
	SANDWICH	FRUIT	DRINK	SWEETS
MON				
TUES				
WED				
THUR				
FRI				

Problem 17

Number 1 2 3

Level 3

Investigate number patterns in nature. Number patterns can be seen in the arrangement of petals of flowers and in the way leaves are formatted. They can also be found in sea shells, fruit and seeds. Examine objects that are found in nature (cross-sections of segments in fruits and nuts, fallen branches from trees, the placement of leaves, buds and flowers on branches) to discover patterns and then sort the objects into groups or categories according to criteria.



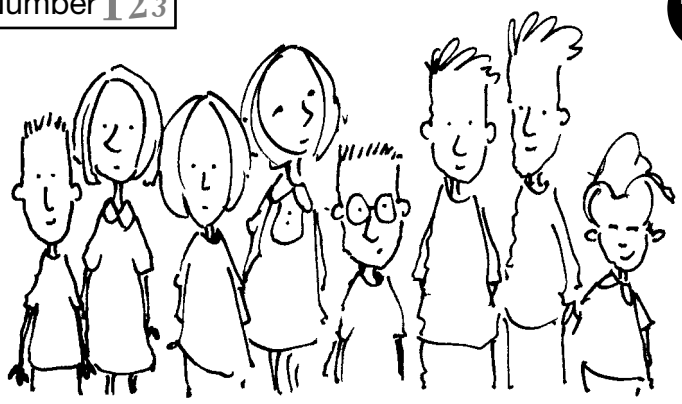
Problem 18

Space

Number 1 2 3

Level 3

How many students could fit into your classroom if all the students were standing and the classroom had no furniture in it?



Answers to Task Cards

Problem 1

Make sure that the students have access to many different chairs for this problem.

Problem 2

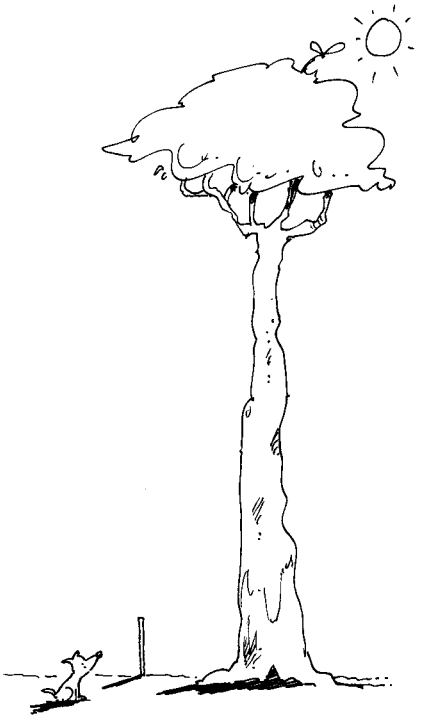
Suggestion 1: students can spread the nails out as flat as possible and then count the number over one small section.
Suggestion 2: students could weigh the whole packet and then weigh 10 nails.

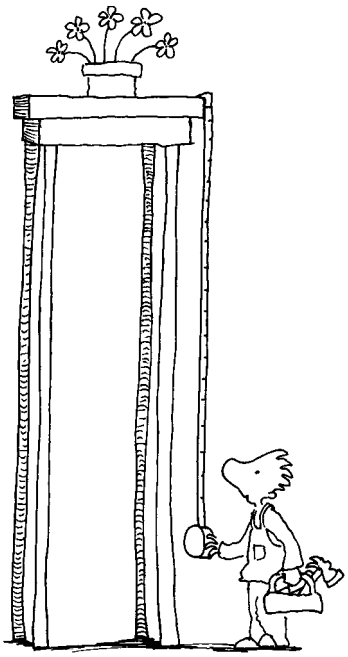
Problem 3

Students can measure the height and width of the wall to find its area in square metres. They then calculate how many bricks to a square metre by measuring out 1 square metre and marking it in chalk. Multiply the number of bricks up by the number of bricks across. This answer is multiplied by the total number of square metres.

Problem 4

Students must have a clear definition of a 3D shape. They should decide how to split the playground into sections. The shapes are then counted in one section and multiplied by how many sections there are.





Problem 5

Students must decide on which balls to use and the size of the box. If they chose a small box encourage them to then work out how many balls it would take to fill a washing machine box; a large fridge box; a shipping container. Answers will vary according to the size of the balls and the boxes.

Problem 6

Children need to have access to empty (clean) drink cans and scales.

Problem 7

Have students list the essential things to remember before they start. They could draw two outlines of their present bedroom on grid paper, then in one draw their furniture etc as near to scale as they can manage. They could then cut out the 'furniture' so this can be moved freely around their new room (ie the other outline).

Problem 8

Before starting, the students should decide whether a survey of one class would be representative—or one lower primary class and one upper primary class.

Problem 9

Answers will vary.
Suggestion: Calculate how many hairs can be found in one square centimetre; how many square centimetres cover an average head. Then calculate the total number of hairs.

Problem 10

Answers will vary.
Each student should draw and cut out many squares. Encourage them to think beyond straight lines only.

Problem 11

One child can sit cross-legged on the playground and his/her shape outlined in chalk. Then students work out how many will fit into the given area. This can be done using number strategies or by making a paper pattern of the outline and fitting it into the given dimensions.

Problem 12

Designate the desired areas, eg reading area, computer area, teacher's desk etc. Students should list all the essential points before they begin, eg lighting, ventilation, power points, boards, display areas etc.

Problem 13

Each student (or small group of students if this is to be a group effort) will need a dental floss container. Answers will vary depending on what the students regard as an average length of floss per person per day. Average length will then be multiplied by four to find the family's usage per day. The daily usage is divided into the total length of floss in the container.

Problem 14

Students should decide on how many windows there are, and which ones have to be measured. They could create square metres out of newspaper. These would be used to measure approximate sizes as they move around the school. The sizes would then be added, or multiplied and added, according to the method chosen.

Problem 15

There will be a variety of methods and answers. There should be coins of each denomination available to the students.

Problem 16

Before beginning discuss the buying patterns of the class. Would a one day sample be fair? List the various methods by which the information could be gathered, eg each child to keep a record of his/her own purchases; price a sample lunch and then work out how many lunches are bought etc. The methods will vary according to type of canteen, goods sold etc.

Problem 17

Have students work in twos or threes. Allow plenty of time for them to explore the school environs. Encourage them to bring fruit, nuts, flowers etc to school. These can be dissected and examined for patterns.

Problem 18

Answers will vary according to the size of the classroom (and the size of the children!).