UNIT P10 ■
Problem Solving
Upper Primary

Blake's Topic Bank

Problem Solving
Open-Ended Problem Solving

by **Sharon Shapiro**

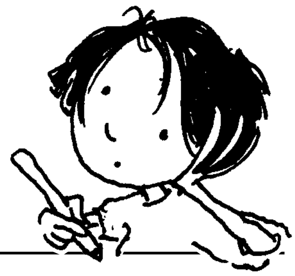
This unit contains:

- Teaching notes
- 3 teaching examples
- 1 BLM
- 18 task cards
- Answers

Problem Solving
Open-Ended Problem Solving

Sharon Shapiro

Upper Primary



The Problem Solving Process

It is important that students follow a logical and systematic approach to their problem solving. Following these four steps will enable students to tackle problems in a structured and meaningful way.

Step 1: Understanding the problem

- ❖ Encourage students to read the problem carefully a number of times until they fully understand what is wanted. They may need to discuss the problem with someone else or rewrite it in their own words.
- ❖ Students should ask internal questions such as, what is the problem asking me to do, what information is relevant and necessary for solving the problem.
- ❖ They should underline any unfamiliar words and find out their meanings.
- ❖ They should select the information they know and decide what is unknown or needs to be discovered. They should see if there is any unnecessary information.
- ❖ A sketch of the problem often helps their understanding.

Step 2: Students should decide on a strategy or plan

Students should decide how they will solve the problem by thinking about the different strategies that could be used. They could try to make predictions, or guesses, about the problem. Often these guesses result in generalisations which help to solve problems. Students should be discouraged from making wild guesses but they should be encouraged to take risks. They should always think in terms of how this problem relates to other problems that they have solved. They should keep a record of the strategies they have tried so that they don't repeat them.

Some possible strategies include:

- ❖ Drawing a sketch, graph or table.
- ❖ Acting out situations, or using concrete materials.
- ❖ Organising a list.
- ❖ Identifying a pattern and extending it.
- ❖ Guessing and checking.
- ❖ Working backwards.
- ❖ Using simpler numbers to solve the problem, then applying the same methodology to the real problem.
- ❖ Writing a number sentence.
- ❖ Using logic and clues.
- ❖ Breaking the problem into smaller parts.

Step 3: Solving the problem

- ❖ Students should write down their ideas as they work so they don't forget how they approached the problem.
- ❖ Their approach should be systematic.
- ❖ If stuck, students should reread the problem and rethink their strategies.
- ❖ Students should be given the opportunity to orally demonstrate or explain how they reached an answer.

Step 4: Reflect

- ❖ Students should consider if their answer makes sense and if it has answered what was asked.
- ❖ Students should draw and write down their thinking processes, estimations and approach, as this gives them time to reflect on their practices. When they have an answer they should explain the process to someone else.
- ❖ Students should ask themselves 'what if' to link this problem to another. This will take their exploration to a deeper level and encourage their use of logical thought processes.
- ❖ Students should consider if it is possible to do the problem in a simpler way.

Teaching Notes

Open-Ended Problem Solving

Open-ended problems challenge a learner's thinking. In looking at open-ended problems, students explore problems that may be answered in a number of ways. However, these problems should always be accurately computed.

These problem solving activities are vitally important, and assist teachers to gauge the level at which their students are achieving, because students respond to challenges at their own level of development. The process that is used is more important than simply achieving the answer. When structuring problems, words such as create, make, design, investigate and explore should be used.

Ask students to develop their own open-ended problems. It is important that students develop their own open-ended problems and solutions as this involves them in developing their thinking and phrasing. If you are helping students solve problems they have set up, model your working as you solve them, as students will learn from your approach to their problems.

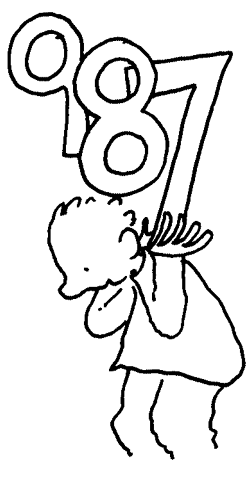
There are some specific skills and strategies that are useful when working with this approach.

Using numbered or labelled counters

When problems become involved, using numbered or labelled counters can help students visualise a problem and its solutions. The counters can be easily manipulated and altered and other combinations found. Changes can easily be made if work is incorrect. When the correct solution is found, it can be written down.

Trying different combinations of numbers

For example, if attempting to solve a problem in which you are to determine the largest product possible using five different numbers, start by using the largest three digit number and then the largest two digit number to see the different answers. (See also teaching example 1.)



Using the largest three-digit number:

$$\begin{array}{r} 987 \\ \times\ 65 \\ \hline 64\,155 \end{array}$$

Using the largest two-digit number:

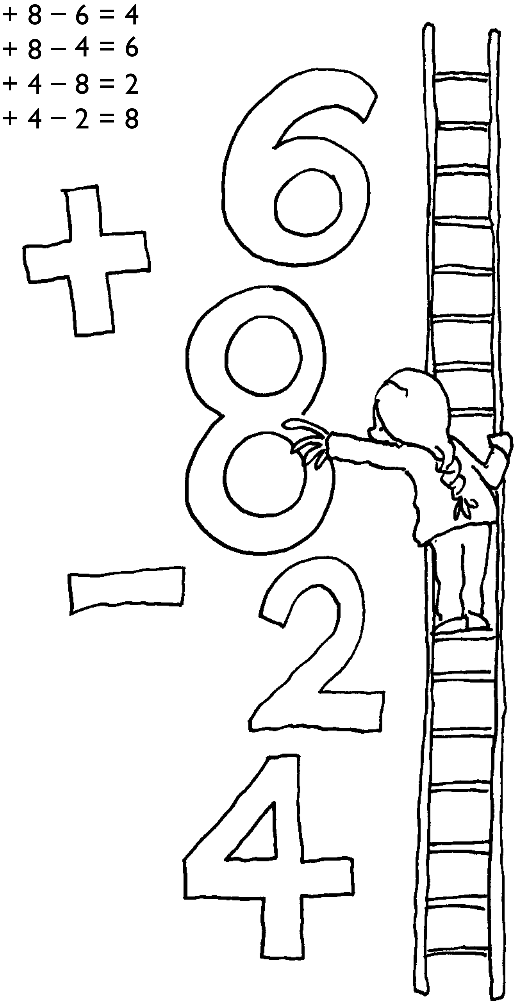
$$\begin{array}{r} 642 \\ \times\ 98 \\ \hline 62\,916 \end{array}$$

Continue working to find as many solutions as possible

This involves students in working to find more than one answer, and manipulating figures in order to look at them differently. Students are thereby encouraged to become involved with genuine mathematical solutions.

eg What can we do with 2 4 6 8 = + – ?

2 + 8 – 6 = 4
2 + 8 – 4 = 6
6 + 4 – 8 = 2
6 + 4 – 2 = 8



EXAMPLE 1

Explore different arrangements of the digits 2, 4, 6, 8 and 9 so that you create the largest product possible, using only one multiplication sign (×). Each numeral must be used, and can be used only once.

Understanding the problem

WHAT DO WE KNOW?

There are different arrangements of digits, one of which will enable us to reach the highest possible product.

WHAT DO WE NEED TO FIND OUT?

Questioning: What are the digits? How should they be arranged?

COMMUNICATING A SOLUTION

Begin by using the largest three digit number you can make:

$$\begin{array}{r} 986 \\ \times\ 42 \\ \hline 41\,412 \end{array}$$

Now try using the largest two digit number.

$$\begin{array}{r} 642 \\ \times\ 98 \\ \hline 62\,916 \end{array}$$

As we multiply numbers together we see that 800 × 90 or 900 × 80 will give you 72 000.

So: or:

$$\begin{array}{r} 842 \\ \times\ 96 \\ \hline 80\,832 \end{array} \qquad \begin{array}{r} 962 \\ \times\ 84 \\ \hline 80\,808 \end{array}$$

While:

$$\begin{array}{r} 862 \\ \times\ 94 \\ \hline 81\,028 \end{array} \qquad \begin{array}{r} 942 \\ \times\ 86 \\ \hline 81\,012 \end{array}$$

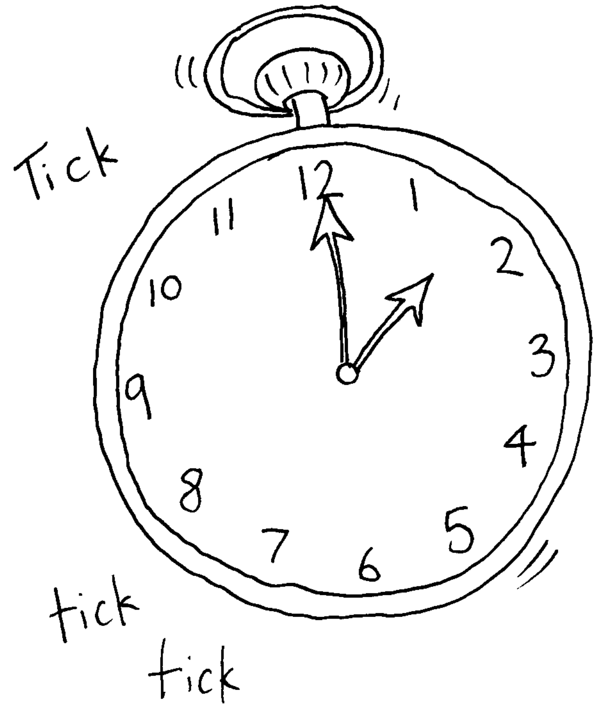
862 × 94 resulted in the largest product.

Reflecting and generalising

The process of trying one solution and then trying others allows students to become familiar with what happens when we multiply different combinations of numbers. This knowledge can then be applied to numerous other problems and will form part of their solutions.

Extension

Repeat the example making the multiplication 3 digits x 3 digits. This will take a lot longer, so allow plenty of time.



EXAMPLE 2

Investigate which combinations of the digits 3, 4, 6, and 7 will create addition problems that have a sum falling between 100 and 120.

Understanding the problem

WHAT DO WE KNOW?

There are different combinations of digits that can be used in addition problems. These will enable us to reach the appropriate total.

WHAT DO WE NEED TO FIND OUT?

Questioning: What combination of digits total between 100 and 120?

COMMUNICATING A SOLUTION

By trying different combinations the following were found to be suitable solutions.

$$\begin{array}{r}43\\ +\,76\\ \hline 119\end{array}\qquad \begin{array}{r}34\\ +\,67\\ \hline 101\end{array}\qquad \begin{array}{r}43\\ +\,67\\ \hline 110\end{array}\qquad \begin{array}{r}34\\ +\,76\\ \hline 110\end{array}$$

$$\begin{array}{r}37\\ +\,64\\ \hline 101\end{array}\qquad \begin{array}{r}36\\ +\,74\\ \hline 110\end{array}$$

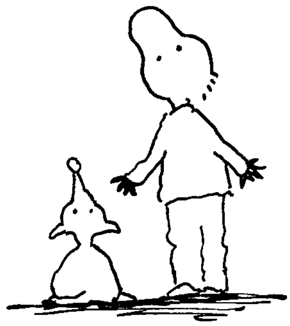
Reflecting and generalising

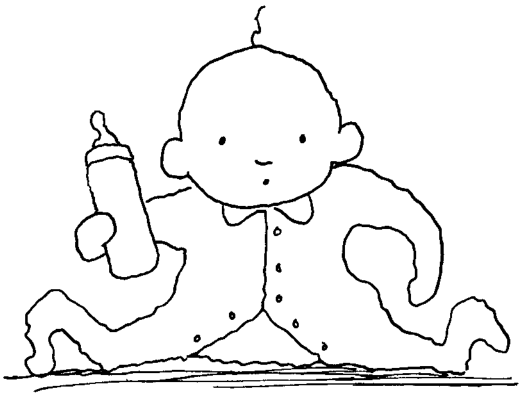
The process of trying one solution and then trying others allows us to become familiar with what happens when we add different combinations of numbers.

This knowledge can then be applied to numerous other problems and will form part of their solutions.

Extension

The same problem can be given using subtraction, for example: using the digits 2, 4, 6, and 7 which combinations when subtracted will have a difference of between 20 and 30?





Example 3

There are three children. The product of their ages is 20. How old might they be? (A 'child' is counted as being under 18, and the ages in this problem are calculated in whole years.)

Understanding the problem

What do we know?

There are three whole numbers. The three numbers are multiplied to make the product 20. There are different arrangements of digits that will enable us to reach the appropriate total.

What do we need to find out?

Questioning: How old are the children? Which numbers were multiplied together?

Communicating a solution

There are groups of three numbers that can be multiplied together to make 20. (We are finding the factors of 20.)

1, 2, 10
1, 4, 5
2, 2, 5
1, 1, 20

The answer 1, 1, 20 is not appropriate, as a 'child' must be under eighteen, so we are left with three answers, all of which could be correct.

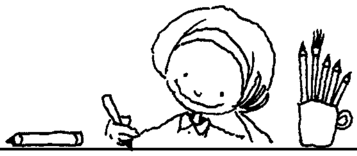
Reflecting and generalising

The process of finding a number of solutions or all the solutions encourages students to attempt different methods. Depending on their mathematical ability, they may or may not understand that they are looking at factors.

Extension

What if there were four children and the product of their ages was 36? What is the lowest possible product of the ages of two adults? Of three adults?

BLM Open-Ended Problem Solving



★ Understanding the problem

List what you know from reading the problem

★ What do you need to find out?

What questions do you have? What are you uncertain about? Is there any unfamiliar or unclear language?

★ Planning and communicating a solution

Start by reading the problem. Decide what you are being asked to do. Decide where to start. Try to work methodically, thinking all the way through one part at a time. Try to find as many solutions as possible. Have you found all the solutions? Can you see a pattern?

★ Reflecting and generalising

Did the strategy work as planned? Will you be able to apply this method of problem solving to other similar problems? Would a different method have worked better for you for this problem?

★ Extension

How can this strategy be applied to more complicated problems involving additional factors?

Problem 1 Space

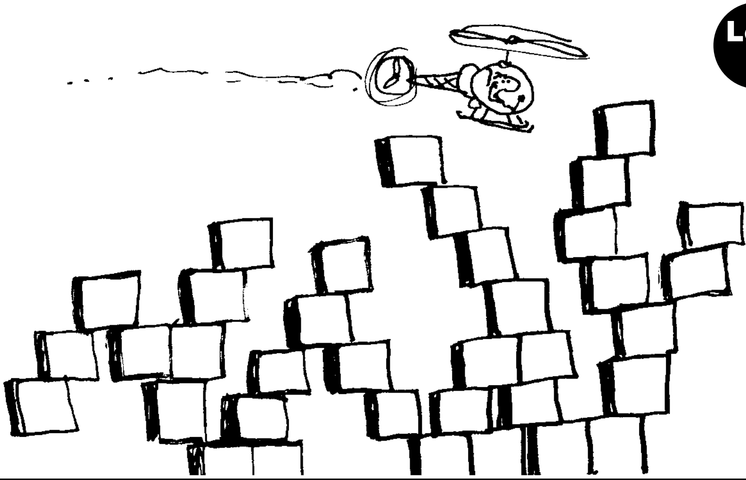
Level 1

Write everything that you know about a rectangle.

Problem 2 Space

Level 1

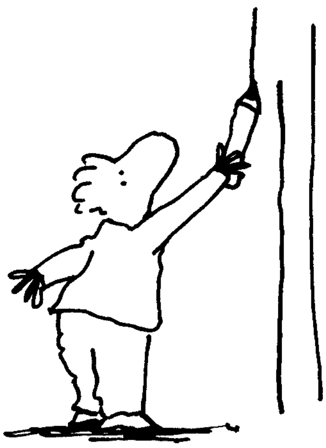
Explore how many ways you can create a building using 36 cubes.



Problem 3 Measurement

Level 1

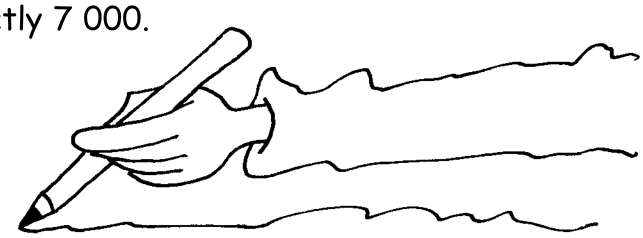
The total length of three lines is 16 cm. Each line is more than 3 cm long but less than 12 cm. How many different sets of lines can you draw that add up to this total length? (Only use whole centimetres.)



Problem 4 Number 1 2 3 Level 1

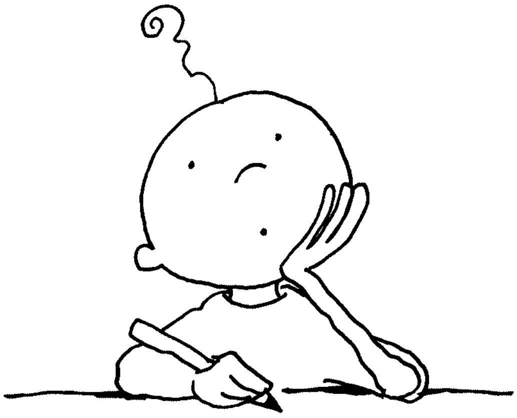
Write five different digits in the boxes so that the product is as close to 7 000 as possible, but not exactly 7 000.

$$\begin{array}{r} \square\ \square\ \square\ \square \\ \times\ \square \\ \hline \end{array}$$



Problem 5 Number 1 2 3 Level 1

How many equations can you make using the numbers 3, 2, 6, 1, and the symbols =, +, ÷? You can only used each numeral once in any equation, but you may use the signs more than once.

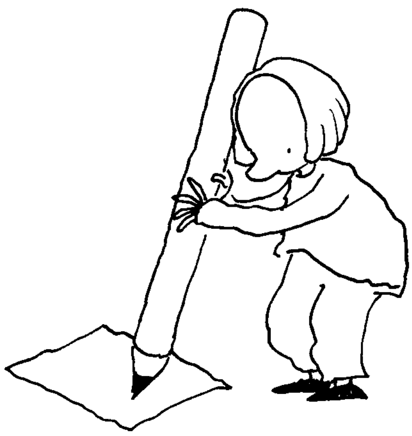


Problem 6 Number 1 2 3 Level 1

Fill in the boxes to make the answer true:

$$\begin{array}{r} 7\ \square\ 9 \\ +\ \square\ 5\ \square \\ \hline \square\ 3\ 4\ 5 \end{array}$$

Is there more than one solution?

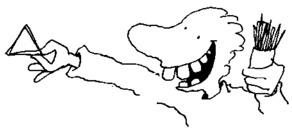


Problem 7 Space

Level 2

Create as many triangles as you can using five toothpicks. How many can you make with 7, 9, 11 or 13 toothpicks? How many can you make with 21? Is there a pattern? Draw them below as you make them.

What happens if there is an even number of toothpicks?



Problem 8 Space

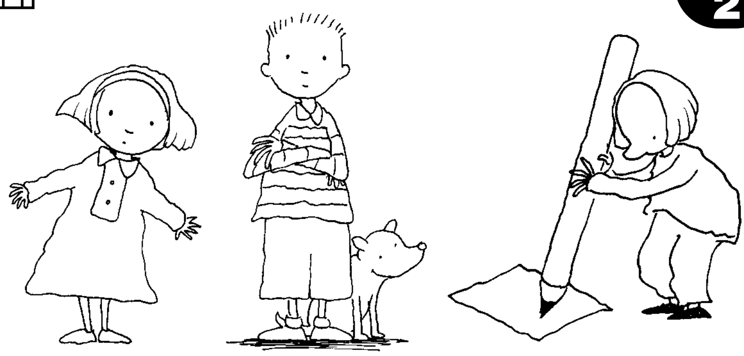
Level 2

Take twelve 1 cm cubes. How many different rectangular prisms can you make using all twelve cubes? Draw them. Is the volume the same for each prism?

Problem 9 Measurement

Level 2

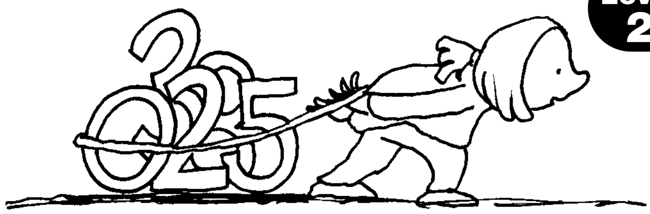
The average height of three people is 170 cm. If you are one of these people, who might be the other two people included in the average?



Problem 10

Number 1 2 3

Level 2



Use zero and the numbers 2, 3, 5, 8, and 9 to make a sum that gives the greatest possible difference.

$$\begin{array}{r} \square\square\square \\ -\ \square\square\square \\ \hline \end{array}$$

(The larger number you make should be at the top. You can't use the zero in the hundreds place.)

Problem 11

Number 1 2 3

Level 2

By using the digits from 0 to 9 in the algorithms below, find (a) the largest possible number and (b) the smallest possible number you can make. (You may not use the same digit more than once in any algorithm.)

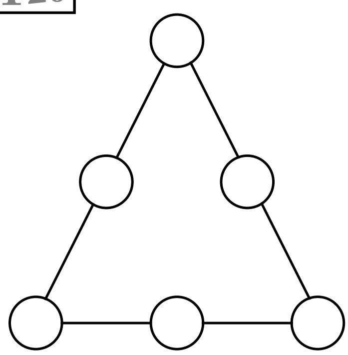
$$\begin{array}{r} \square\square \\ \times\ \square\square \\ \hline \end{array} \qquad \begin{array}{r} \square\square \\ \times\ \square\square \\ \hline \end{array} \qquad \begin{array}{r} \square\square \\ \times\ \square\square \\ \hline \end{array} \qquad \begin{array}{r} \square\square \\ \times\ \square\square \\ \hline \end{array}$$

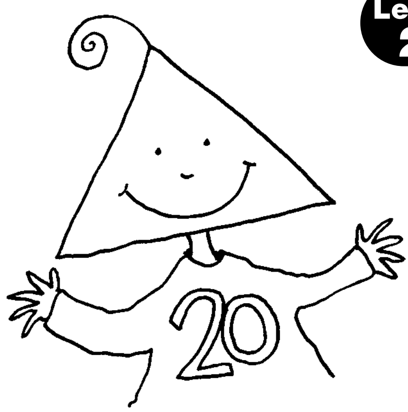
Problem 12

Number 1 2 3

Level 2

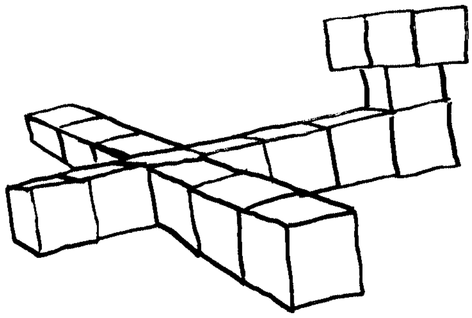
Using the even numbers 2, 4, 6, 8, 10 and 12, fill in the circles so that each side of the triangle adds to 20.





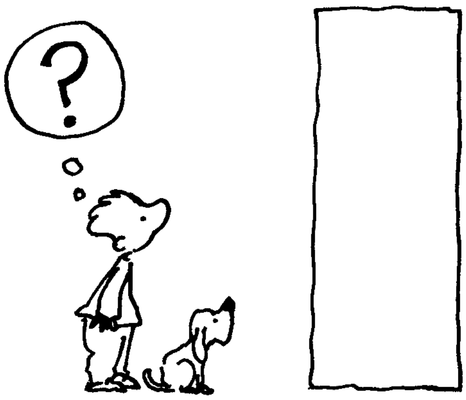
Problem 13 Space Measurement Level 3

Take 24 centicubes. Build four different models using all the cubes. Draw them. Write a statement about the volume of each model.



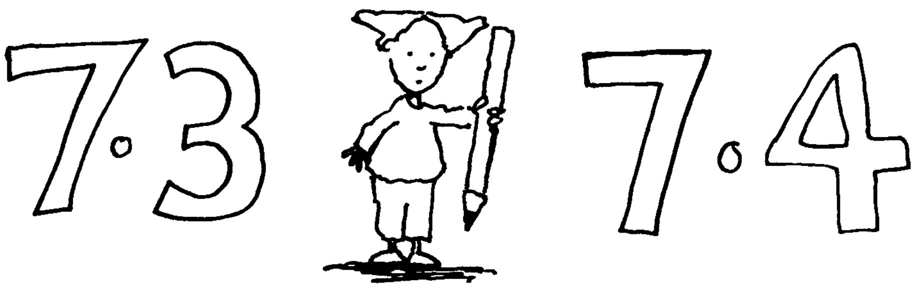
Problem 14 Measurement Level 3

A rectangle has an area of 24 cm^2. What might be the length of its sides?



Problem 15 Number 123 Level 3

Write down fifteen numbers that could fall between 7·3 and 7·4.



Problem 16

Number 1 2 3

Level 3

Find four digits to multiply that will have a product that is as close to 789 as possible. Use a calculator.

$$\begin{array}{r}\square\square\\ \times\ \square\square\\ \hline\end{array}\quad\begin{array}{r}\square\square\\ \times\ \square\square\\ \hline\end{array}\quad\begin{array}{r}\square\square\\ \times\ \square\square\\ \hline\end{array}\quad\begin{array}{r}\square\square\\ \times\ \square\square\\ \hline\end{array}\quad\begin{array}{r}\square\square\\ \times\ \square\square\\ \hline\end{array}\quad\begin{array}{r}\square\square\\ \times\ \square\square\\ \hline\end{array}$$

Problem 17

Number 1 2 3

Level 3

Find at least five solutions, using counters numbered from 1 to 9. The total you are aiming for is 999.

$$\begin{array}{r}\square\square\square\\ +\ \square\square\square\\ \hline 9\ 9\ 9\end{array}\quad\begin{array}{r}\square\square\square\\ +\ \square\square\square\\ \hline 9\ 9\ 9\end{array}\quad\begin{array}{r}\square\square\square\\ +\ \square\square\square\\ \hline 9\ 9\ 9\end{array}$$

$$\begin{array}{r}\square\square\square\\ +\ \square\square\square\\ \hline 9\ 9\ 9\end{array}\quad\begin{array}{r}\square\square\square\\ +\ \square\square\square\\ \hline 9\ 9\ 9\end{array}\quad\begin{array}{r}\square\square\square\\ +\ \square\square\square\\ \hline 9\ 9\ 9\end{array}$$

Problem 18

Number 1 2 3

Level 3

Solve each number sentence by replacing the gaps with the same number. (The number is different for each number sentence.)

[(_ + _) × _] - _ = 28

_ + [(_ - _) + _] = 6

(_ × _) + (_ - _) - _ = 20

(_ + _) ÷ _ - _ = 0

(_ + _) × _ = 242

(_ + _) × _ = 450

Answers to Task Cards

Problem 1

A rectangle has four sides, four angles, straight sides, two diagonals, two axes of symmetry. All its angles are right angles. Opposite sides are equal. It is a 2D plane shape.

Problem 2

Teacher to check all models. Students should be encourage to be creative when building their models.

Problem 3

4 cm, 4 cm, 8 cm
4 cm, 5 cm, 7 cm
4 cm, 6 cm, 6 cm
5 cm, 6 cm, 5 cm

Problem 4

Some solutions could be:

$3456 \times 2 = 6912$

$3470 \times 2 = 6940$

$3498 \times 2 = 6996$

$3501 \times 2 = 7002$

Encourage students to make a list of other ‘close’ solutions.

Problem 5

$3 + 2 + 1 = 6$

$3 \times 2 = 6$

$3 \times 2 \times 1 = 6$

$6 \div (3 \times 1) = 2$

$6 \div (2 \times 1) = 3$

$6 \div (2 \times 3) = 1$

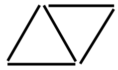
$6 - (2 + 3) = 1$

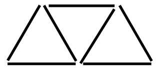
Problem 6

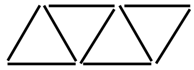
$$\begin{array}{rcccc} & & 7 & \boxed{8} & 9 \\ + & & \boxed{5} & 5 & \boxed{6} \\ \hline & \boxed{1} & 3 & 4 & 5 \end{array}$$

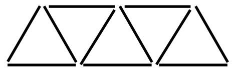
There is only one solution.

Problem 7


5 toothpicks / 2 triangles

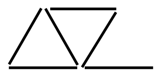

7 toothpicks / 3 triangles


9 toothpicks / 4 triangles

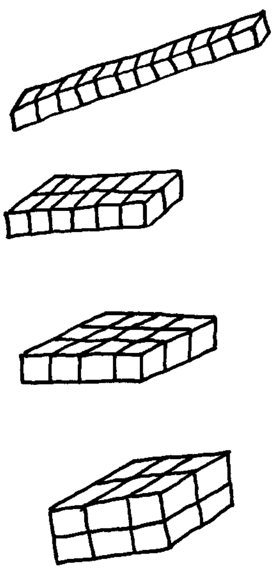

11 toothpicks / 5 triangles

As the odd number of toothpicks increases, a new triangle is added each time. For 21 toothpicks there will be $(21 - 1) \div 2 = 10$ triangles.

If there is an even number of toothpicks, there will always be one side left over in this pattern.



Problem 8



The volume is the same for each prism.

Problem 9

In most cases, the student will be much shorter than 170 cm, so the other two people will have to be taller than 170 cm. Verify that the student worked out that the total heights would be 510 cm. It would be a valuable exercise to discuss the answers to this problem as a class.

Problem 10

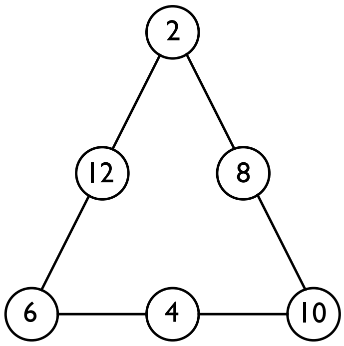
$$\begin{array}{r} 985 \\ -\ 203 \\ \hline 782 \end{array}$$

Problem 11

$$\begin{array}{r} 96 \\ \times 87 \\ \hline 8352 \end{array}$$

$$\begin{array}{r} 23 \\ \times 10 \\ \hline 230 \end{array}$$

Problem 12



Problem 13

Teacher to check models. The volume of each model will be 24 cm^3 as 24 × 1 cm^3 blocks were used.

Problem 14

1 cm × 24 cm
2 cm × 12 cm
3 cm × 8 cm
4 cm × 6 cm
2·5 cm × 9·6 cm
2·4 cm × 10 cm
3·2 cm × 7·5 cm
1·5 cm × 16 cm

Complete list from students' answers.

Problem 15

Teacher to check answers.

Any decimal number which starts with 7·3 and is followed by one or more digits is correct, eg 7·31, 7·3008.

Problem 16

Some solutions are:

65 × 12 = 780
17 × 46 = 782
19 × 42 = 798
21 × 38 = 798
47 × 17 = 799

Problem 17

Any pairs using the combinations for 9 will work, eg 1 + 8, 2 + 7, 3 + 6, 4 + 5. Some possible solutions would be:

$$\begin{array}{r} 123 \\ +\ 876 \\ \hline 999 \end{array} \qquad \begin{array}{r} 234 \\ +\ 765 \\ \hline 999 \end{array}$$

$$\begin{array}{r} 537 \\ +\ 462 \\ \hline 999 \end{array}$$

Problem 18

[(4 + 4) ×4] − 4 = 28
3 + [(3 − 3) + 3] = 6
(5 + 5) + (5 − 5) − 5 = 20
(2 + 2) ÷ 2 − 2 = 0
(11 + 11) ×11 = 242
(15 + 15) ×15 = 450